Wednesday, April 09, 2014
7:48 AM

Now that we have the Feynman rules for QCD down, we can use them to investigate aspects of QCD:

- Why do we find only color singlet bound states (mesons, baryons)?
- If the strong force is so strong, why is it swamped by EM at large distances?

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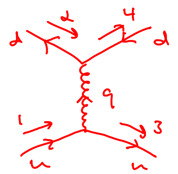
Okay, so let's investigate why hadrons must be color singlets.

Example: Mesons ($q_1\bar{q}_2$)

The idea is to see if the strong force is attractive (hence bound states) or repulsive (no bound states).
The answer we expect is:

a) For colorless singlet combination $\rightarrow$ attractive
b) For colorful octet combination $\rightarrow$ repulsive

At leading order:



$u + \bar{d} \rightarrow u + \bar{d}$

Not matrices in spin-space!

$$M = i\,\bar{u}(3)c(3)^{\dagger}\left(-i\frac{g_s}{2}\lambda^{\alpha}\gamma^{\mu}\right)u(1)c(1)\left[-\frac{i\eta_{\mu\nu}\delta^{\alpha\beta}}{q^2}\right]\bar{v}(2)c(2)^{\dagger}\left(-i\frac{g_s}{2}\lambda^{\beta}\gamma^{\nu}\right)v(4)c(4)$$

after cancelling $(2\pi)^4\delta^4(P_{tot\,in} - P_{tot\,out})$ and $\times i$;

$$= \underbrace{-\frac{g_s^2\eta_{\mu\nu}}{q^2}\bar{u}(3)\gamma^{\mu}u(1)\bar{v}(2)\gamma^{\nu}v(4)}_{\text{Exactly what we would have gotten from QED w/ } g_e \rightarrow g_s \text{ for example } e^+ + \mu^- \rightarrow e^+ + \mu^- \text{ which is attractive !!}}\ \underbrace{\left[\left(c(3)^{\dagger}\lambda^{\alpha}c(1)\right)\left(c(2)^{\dagger}\lambda^{\alpha}c(4)\right)\right]\times\frac{1}{4}}_{\text{color "factor"}}$$

Color sandwiches (giving numbers)

In a sense you can model the interaction w/ a potential $V(r)$ and we are calculating $\langle\psi_f|V(r)|\psi_i\rangle$.
But we know that attractive vs. repulsive is determined by the sign of $V(r)$.

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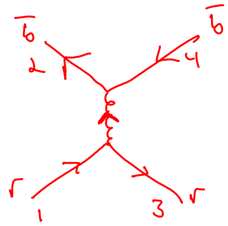
So the new step is evaluating the color factor f:

If the $u\bar{d}$ were in a colorful "octet" state, e.g. $r\bar{b} = \frac{1}{\sqrt{2}}(|1\rangle + i|2\rangle)$ then:



$$\Rightarrow \quad c(1) = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad c(2) = \begin{pmatrix}0\\1\\0\end{pmatrix} \quad c(3) = \begin{pmatrix}1\\0\\0\end{pmatrix} \quad c(4) = \begin{pmatrix}0\\1\\0\end{pmatrix}$$

In the above we must have the color assignments due to overall color conservation. So in this case the gluon does not change the color at each vertex, e.g. it could be $|8\rangle = \frac{1}{\sqrt{6}}(r\bar{r} + b\bar{b} - 2g\bar{g})$.
Gluons can change color but do not have to.

Then: $f = \frac{1}{4}\begin{pmatrix}1 & 0 & 0\end{pmatrix}\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}0 & 1 & 0\end{pmatrix}\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix} = \frac{1}{4}\lambda^\alpha_{11}\lambda^\alpha_{22} = \frac{1}{4}\sum_{\alpha=1}^{8}\lambda^\alpha_{11}\lambda^\alpha_{22}$

$$\lambda^1 = \begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix} \quad \lambda^2 = \begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix} \quad \lambda^3 = \begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix} \quad \lambda^4 = \begin{pmatrix}0&0&1\\0&0&0\\1&0&0\end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix} \quad \lambda^6 = \begin{pmatrix}0&0&0\\0&0&1\\0&1&0\end{pmatrix} \quad \lambda^7 = \begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix} \quad \lambda^8 = \frac{1}{\sqrt{3}}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}$$

Finally: $f = \frac{1}{4}\left(0 + 0 - 1 + 0 + 0 + 0 + 0 + \frac{1}{3}\right) = -\frac{1}{6}$

Recall that the only difference between this amplitude and the attractive e+m amplitude is this color factor. Since it is negative this implies a repulsion between a quark and anti-quark in a color octet state.

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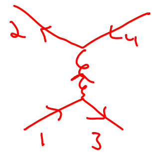
If we used the colorless singlet $|g\rangle = \underbrace{\frac{1}{\sqrt{3}}\left(r\bar{r} + b\bar{b} + g\bar{g}\right)}$ then:

Note: This is $\underline{\underline{\text{not}}}$ $(r+b+g)(\bar{r}+\bar{b}+\bar{g})$



$$\Rightarrow \begin{cases} c(1)c(2) = \frac{1}{\sqrt{3}}\left[\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix} + \begin{pmatrix}0\\0\\1\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}\right] \\ c(3)c(4) = \frac{1}{\sqrt{3}}\left[\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix} + \begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix} + \begin{pmatrix}0\\0\\1\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}\right] \end{cases}$$

Then $f = \frac{1}{4}(c_3^\dagger \lambda^\alpha c_1)(c_2^\dagger \lambda^\alpha c_4) = \frac{1}{4}\frac{1}{\sqrt{3}} c_3^\dagger \lambda^\alpha \left[\begin{pmatrix}1\\0\\0\end{pmatrix}(100) + \begin{pmatrix}0\\1\\0\end{pmatrix}(010) + \begin{pmatrix}0\\0\\1\end{pmatrix}(001)\right] \lambda^\alpha c_4$

$$= \frac{1}{4}\frac{1}{\sqrt{3}}\frac{1}{\sqrt{3}}\Big[(100)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}(100)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix} + (010)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}(100)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix} + (001)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix}(100)\lambda^\alpha\begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$+ (100)\lambda^\alpha\begin{pmatrix}0\\1\\0\end{pmatrix}(010)\lambda^\alpha\begin{pmatrix}1\\0\\0\end{pmatrix} + \cdots \quad \cdots$$

$$\vdots \qquad \Big]$$

$$= \frac{1}{12}\lambda^\alpha_{ij}\lambda^\alpha_{ji} = \frac{1}{12}\underbrace{\mathrm{Tr}(\lambda^\alpha\lambda^\alpha)}_{16} = \frac{4}{3}$$

Notice that for the singlet configuration the color factor is positive and hence the strong interaction between a quark and anti-quark in this state is attractive!

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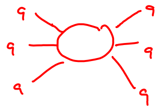
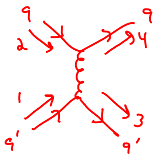
Example Baryons. $(q_1 q_2 q_3)$

To show that the force between the three quarks in a baryon is attractive when they form a color singlet (and repulsive otherwise) is considerably more tedious.

You might think to evaluate  , but this is complicated and a bit misleading.

The problem is that we might get some "overall" attraction but with a repulsion between two of the quarks.

To analyze this it is easier to first consider pairs of quarks;



$$\Rightarrow \quad \mathcal{M} = \underbrace{-\frac{g_s^2}{4}\frac{1}{q^2}[\bar{u}(3)\gamma^\mu u(1)][\bar{u}(4)\gamma_\mu u(2)]}_{\text{Same as repulsive electron-muon scattering.}}(c_3^\dagger \lambda^\alpha c_1)(c_4^\dagger \lambda^\alpha c_2)$$

To evaluate the color factor we need to consider combinations of color assignments for q, q'.

For $c\bar{c}'$ we had $3 \otimes \bar{3} = \underset{\text{octet}}{8} \oplus \underset{\text{singlet}}{1}$

For cc' we get $3 \otimes 3 = \underset{\text{triplet}}{\bar{3}} \oplus \underset{\text{sextet}}{6}$ where $\overbrace{\left\{\frac{1}{\sqrt{2}}(rb-br), \frac{1}{\sqrt{2}}(gr-rg), \frac{1}{\sqrt{2}}(bg-gb)\right\}}^{\text{anti-symmetric under } c \leftrightarrow c'}$ = triplet

$\underbrace{\left\{rr, bb, gg, \frac{1}{\sqrt{2}}(rb+br), \frac{1}{\sqrt{2}}(gr+rg), \frac{1}{\sqrt{2}}(gb+bg)\right\}}_{\text{symmetric under } c \leftrightarrow c'}$ = sextet

Compare w/ spin-$\frac{1}{2}$ in 3D

$2 \otimes 2 = 1 \oplus 3$

$\frac{1}{2}$ $\frac{1}{2}$ scalar vector

The color factors for any combination of the two types are:

$f_{trip} = -\frac{2}{3} \quad \Rightarrow$ attractive

$f_{sex} = \frac{1}{3} \quad \Rightarrow$ repulsive

Now if we combine 3 quarks we have: $3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$

10: totally symmetric decuplet, e.g. $\frac{1}{\sqrt{6}}(rgb + bgr + rbg + brg + grb + gbr)$
rrr, etc.

$8 \oplus 8$: mixed symmetry octets
e.g. $\frac{1}{\sqrt{2}}(rb-br)b$
$\frac{1}{\sqrt{2}}g(rb-br)$
etc.

1: totally antisymmetric singlet
e.g. $\frac{1}{\sqrt{6}}(rgb - rbg - grb + brg + gbr - bgr)$

Only in the totally anti-symmetric singlet configuration is the force between any two pairs of quarks in the baryon attractive.

Therefore to reliably predict bound states we need the three quarks in a hadron to form a color singlet.

So the moral of this is that fully attractive interactions which seem reasonable for bound states, require colorless singlet configurations of quarks. This is not a rigorous proof.

On the other hand, the SU(3) symmetry only allows colorful octet gluons.

There are two experimental observations that we should consider: a) we never see free quarks
b) we never see free gluons

Both of these are "explained" by the "color confinement hypothesis", i.e. that any free (or long lived) state in QCD must be in a color singlet configuration. This rules out gluons and single quarks, but obviously allows mesons and baryons!

Now proving this hypothesis is one of the great outstanding problems in particle physics. For reasons we will see when we study renormalization, it has a lot to do with the breakdown of perturbation theory (the QCD coupling $g > 1$) in certain situations, so the Feynman approach is useless. Exact methods like lattice gauge theory, supersymmetry and dualities might eventually shed some light.

For now, let's take this hypothesis as a given and look at some implications.

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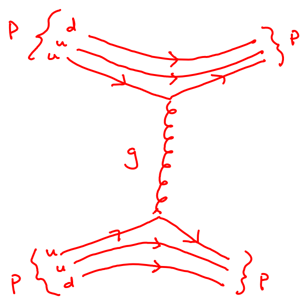
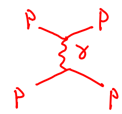
Strong Interactions Seem Weak

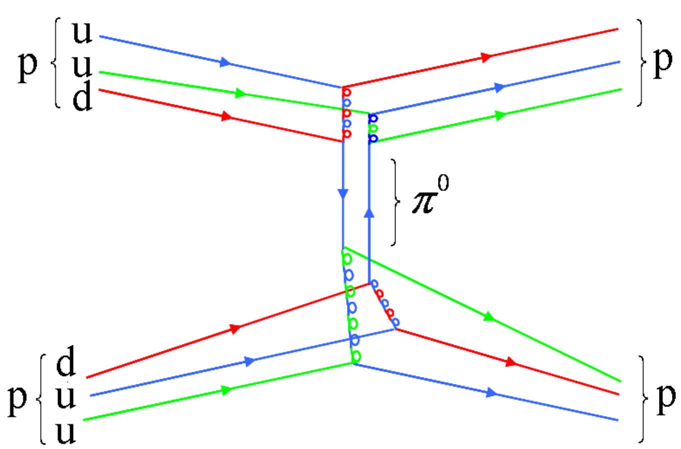
Even though $\alpha_S > \alpha_W > \alpha_E$ and the gluons are massless, QCD manifests itself as a relatively weak force (compared to EM) between two well separated protons.

Consider proton-proton scattering $p+p \to p+p$:

If:  then $\mathcal{O}(\alpha_S^2) > \mathcal{O}(\alpha_E^2)$  !

But there are two problems here. First of all each proton is an overall color singlet, so there QCD interaction is a bit more akin to the Van der Waals interaction between two neutral atoms. Additionally, since they are "well separated" the mediator is reasonably long lived and thus should be a color singlet (by confinement), so it cannot be a gluon.

Okay, so how do 2 protons interact via QCD? Consider one possibility:



There are several important observations.

a) Overall this looks like the two protons exchange a pion, which, as a meson, can happily exist as a color singlet.

b) Compared to the QED process which is $\mathcal{O}(\alpha_E^2)$ this is $\mathcal{O}(\alpha_S^8)$. So this will usually be swamped by the EM version.

c) The attraction between p + n has an almost identical diagram, but in that case no electromagnetic competition.

d) This entire picture is simplified, in part because each proton is really in $\frac{1}{\sqrt{6}}(rgb - rbg - grb + brg + gbr - bgr)$ while the π^0 is in $\frac{1}{\sqrt{3}}(r\bar{r} + b\bar{b} + g\bar{g})$.

Thursday, April 19, 2018
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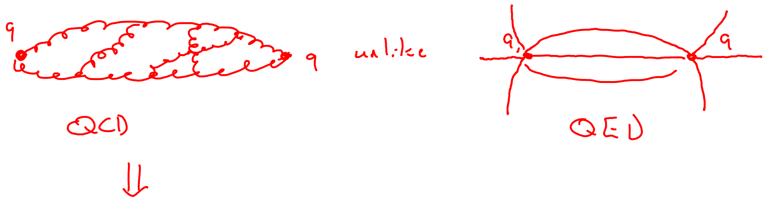
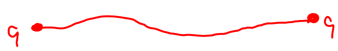
An early strategy (prior to QCD) to explain the absence of free quarks was based on the idea that the force between separated quarks gets confined to a tube connecting them. These color flux tubes may very well occur in QCD in part due to the gluon-gluon attraction:





Studying the dynamics of these fluxtubes ($\sim 10^{-15}$ m) led to problems like tachyons, spin-2 excitations, etc.
Eventually a couple of folks said that if we shrunk these to 10^{-35} m, then the spin-2 state could be the graviton, and by adding supersymmetry we could get rid of the tachyon. Thus was born String Theory.