Weak Sauce

Okay, so we have known a lot about E+M for over 100 years. What makes QCD and Weak interactions harder. For QCD it's that quarks (and gluons) are bound.

The weak interactions are both complicated (think Lagrangians w/ spontaneously broken symmetry!) and very important (think true particle decay!).

First and foremost, they are not that weak, i.e. $\alpha_S > \alpha_W > \alpha_E$ at low energies. Rather, we find that the weak interaction related amplitudes are suppressed because the $W^\pm, Z^0$ are so massive!

We will spend much of today on the ugliness of the weak interactions, and then next time put them to use in order to analyze particle decays.

Wednesday, April 16, 2014
1:14 PM

CKM (the power of theory)

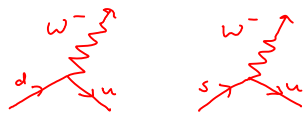
The $W^\pm$ change lepton flavor within a single generation $\begin{pmatrix} \nu_e \\ e \end{pmatrix}\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$

etc.

In a simple world, the $W^\pm$ would do the same for quarks, but alas... time for a history lesson.

$$\begin{pmatrix} u \\ d \end{pmatrix}\begin{pmatrix} c \\ s \end{pmatrix}\begin{pmatrix} t \\ b \end{pmatrix}\begin{matrix} \frac{2}{3} \\ -\frac{1}{3} \end{matrix} \qquad \overset{M}{\longrightarrow}\; \begin{pmatrix} u \\ d \end{pmatrix}\begin{pmatrix} s \\ c \end{pmatrix}\begin{pmatrix} b \\ t \end{pmatrix} \downarrow M$$

In 1963 there were only (u,d,s) quarks known, but to describe the results of experiments Cabibbo realized that both d and s should be related to u and quantified this w/

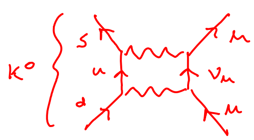


← this guy is changing generation!

$$\underbrace{-\frac{ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)\cos\theta_c}_{\text{dominant contribution}} \qquad -\frac{ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)\sin\theta_c$$

and in fact $\theta_c = 13.5^\circ$

However at least one puzzle remained:



$K^0 \to \mu^+ + \mu^-$

$\Downarrow$

$\mathcal{M} \propto \sin\theta_c \cos\theta_c$ (from vertices)

However experiments showed this to be extremely suppressed, i.e. the K^0 lived far longer than this predicted.

Tuesday, April 24, 2018
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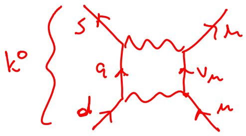
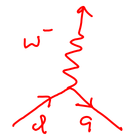
In 1970 Glashow, Iliopoulos and Maiani (GIM) proposed a "cancelling" diagram of the form:



w/ $M \propto -\sin\theta_c \cos\theta_c$ from



$-\frac{ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)(-\sin\theta_c)$ $\quad -\frac{ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)\cos\theta_c$

This new theorized was called the "charm" quark and wasn't directly discovered until the first $c\bar{c}$ production in 1974.

Defining: $d' = d\cos\theta_c + s\sin\theta_c$
$s' = -d\sin\theta_c + s\cos\theta_c$

Then the $W^\pm$ rotate within doublets of the form $\begin{pmatrix} u \\ d' \end{pmatrix}\begin{pmatrix} c \\ s' \end{pmatrix}$

But why? The transformation from d, s to d', s' is clearly a rotation of some kind, but moreover it takes for example a purely d' state and expresses it as a superposition of d and s. Where else in physics do we encounter this type of single state → superposition state behavior? How about measuring non-commuting observables in QM?!

But what non-commuting operators are at play here? The root of the problem is that quarks propagate as eigenstates of the free Dirac Hamiltonian $\hat{H}_0$, but they decay due to interactions associated w/ weak vertex operators $\hat{V}_w$. The key point is that $[\hat{H}_0, \hat{V}_w] \neq 0$, and so eigenstates of one are not simultaneously eigenstates of the other.

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But it gets even better. After GIM theorized the c quark, but a year before it was experimentally found, Kobayashi and Maskawa decided "the more the merrier" so they introduced a third generation of quarks and extended the Cabibo rotation to a 3×3 CKM transformation:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{\approx \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

magnitudes (matrix elements are actually complex)

Actually they had a good reason to do so. They really wanted a truely complex quark rotation matrix, but for a 2×2 it could be made real by redefining quark phases. With a 3×3 though it could be complex and not be rendered real.

The complex element in the CKM is the root of CP-violation in the SM (measured in 1964).

Eventually as of 1995 all of the "theorized" quarks have been produced in collider experiments.

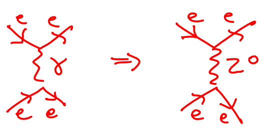
In practice when evaluating Feynman amplitudes w/ vertices

use vertex $-\frac{i g_w}{2\sqrt{2}} \gamma^\mu (1-\gamma^5) V_{high\,low}$

$\begin{pmatrix} q_{high} \\ q_{low} \end{pmatrix}$

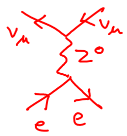
The Z^0 was first theorized in 1958 (Bludman) and put on "solid" theoretical ground by Glashow in 1961 when he unified electromagnetism and the weak interactions into a single electro-weak theory. In 1967 Weinberg and Salaam finished the story by explaining why the interactions appear so different today. Altogether this forms the GWS electroweak theory in which the Higgs mechanism plays a key role.

It took until 1973 to get experimental confirmation of the Z^0. Why so long? Remember the Z^0 is interchangeable w/ γ in any QED diagram,



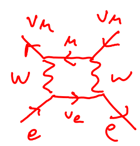
but the $M_{QED} >> M_{weak}$ so the weak process is always overshadowed.

However the reverse is not true and there are some weak diagrams w/ Z^0 that cannot by replaced w/ γ, i.e.



This process was observed at CERN in 1973.

You might argue that they could have seen



, but this is 4th order compared to Zo and would have come w/ a much smaller amplitude.

Using the Z^0: Since the Z^0 doesn't change flavor it acts much like the γ in QED. In fact we don't even get any CKM nonsense (since that is only relevant for flavor change).

However the weak interactions always find a way to be a pain, so here is how the Z^0 acts like a problem child.

Recall the usual vertex factor is $-\frac{ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$ modulo CKM factors or quark composite corrections.

Well the Z^0 vertex factor takes a different form for each flavor:

$-\frac{ig_z}{2}\gamma^\mu(c_V^f - c_A^f\gamma^5)$ where

f	c_V		c_A
ν_e, ν_μ, ν_τ	$\frac{1}{2}$	$= \frac{1}{2}$	$\frac{1}{2}$
e, μ, τ	-0.0806	$= -\frac{1}{2} + 2\sin^2\theta_w$	$-\frac{1}{2}$
u, c, t	0.2204	$= \frac{1}{2} - \frac{4}{3}\sin^2\theta_w$	$\frac{1}{2}$
d, s, b	-0.3602	$= -\frac{1}{2} + \frac{2}{3}\sin^2\theta_w$	$-\frac{1}{2}$

This may seem crazy complicated until you realize that everything is controlled by the Weinberg angle $\theta_w = 28.75°$

$$g_w = \frac{g_e}{\sin\theta_w} \qquad g_z = \frac{g_e}{\sin\theta_w \cos\theta_w} \qquad M_w = M_z \cos\theta_w$$

In the end all of this ugliness and the interconnectedness stems from the spontaneous breaking of the electro-weak symmetry of the early universe!