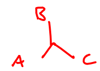
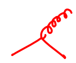
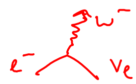
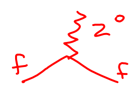
Monday, April 16, 2012
7:57 PM

Now for the (not so) Weak (but fat) Interactions

Let's compare:

	External State Labels	Internal Propagators	Vertex Factors	
ABC	none	$\frac{i}{q^2-m^2c^2}$		$-ig$
QED	$u, \bar{u}, v, \bar{v}$ $\epsilon_\mu, \epsilon^*_\mu$	$\frac{i(\not{q}+mc)}{q^2-m^2c^2}$ $-\frac{ig_{\mu\nu}}{q^2}$		$ig_e\gamma^\mu$
QCD	$uc, \bar{u}c^+, vc, \bar{v}c^+$ $\epsilon_\mu a^\alpha, \epsilon^*_\mu a^{\alpha *}$	$\frac{i(\not{q}+mc)}{q^2-m^2c^2}$ $-\frac{ig_{\mu\nu}\delta^{\alpha\beta}}{q^2}$		$-\frac{ig_s}{2}\lambda^\alpha\gamma^\mu$
				"horrible" (8.43)
				"horribler" (8.44)
Weak	$u, \bar{u}, v, \bar{v}$ $\underbrace{\epsilon_\mu, \epsilon^*_\mu}$	$\frac{i(\not{q}+mc)}{q^2-m^2c^2}$ $\underbrace{\frac{-i(g_{\mu\nu}-q_\mu q_\nu/m^2c^2)}{q^2-m^2c^2}}$		$-\frac{ig_w}{2\sqrt{2}}\underbrace{\gamma^\mu(1-\gamma^5)}_{V-A}$
				$-\frac{ig_z}{2}\gamma^\mu(c_V^f - c_A^f\gamma^5)$

Now have 3-component polarizations since $W^\pm, Z^0$ are massive vector particles.

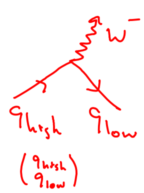
Note this doesn't limit to $q \simeq 0$ propagator, but that is because # of d.o.f. change discontinuously ($3 \to 2$).

f	c_V	c_A
ν_e, ν_μ, ν_τ	$\frac{1}{2}$	$\frac{1}{2}$
e^-, μ^-, τ^-	$-\frac{1}{2}+2\sin^2\theta_w$	$-\frac{1}{2}$
u, c, t	$\frac{1}{2}-\frac{4}{3}\sin^2\theta_w$	$\frac{1}{2}$
d, s, b	$-\frac{1}{2}+\frac{2}{3}\sin^2\theta_w$	$-\frac{1}{2}$

Note: $W^\pm$ are anti-partners
Z^0 is its own anti-particle (like γ)

Also recall that Strong + Weak operators don't commute, so they have different eigenstates.
Quarks are usually created in Strong force eigenstates, but decay by Weak force (so in weak eigenstates).

$$\begin{matrix}\text{high}\\ \text{low}\end{matrix}\ \begin{pmatrix}u\\d\end{pmatrix}\begin{pmatrix}c\\s\end{pmatrix}\begin{pmatrix}t\\b\end{pmatrix} \Rightarrow \begin{pmatrix}u\\d'\end{pmatrix}\begin{pmatrix}c\\s'\end{pmatrix}\begin{pmatrix}t\\b'\end{pmatrix} \quad \text{where} \quad \begin{pmatrix}d'\\s'\\b'\end{pmatrix} = \begin{pmatrix}V_{ud} & V_{us} & V_{ub}\\ V_{cd} & V_{cs} & V_{cb}\\ V_{td} & V_{ts} & V_{tb}\end{pmatrix}\begin{pmatrix}d\\s\\b\end{pmatrix}$$



$$-\frac{ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)V_{high-low}$$

Thursday, April 26, 2018
4:11 PM

Let's talk about the ubiquitous $\gamma^\mu(1-\gamma^5)$ factors for a moment.

Remember that parity is a reflection of an odd-# of coordinates. If we carefully consider how spinors transform under $P(x,y,z) \to (-x,-y,-z)$ we find:

$\psi \to \psi' = \gamma^0 \psi$

which means that

$$\bar{\psi}\psi \to \bar{\psi}'\psi' = \psi'^{\dagger}\gamma^0\psi' = (\gamma^0\psi)^{\dagger}\gamma^0\gamma^0\psi = \psi^{\dagger}\gamma^{0\dagger}\cancelto{I}{\gamma^0\gamma^0}\psi = \psi^{\dagger}\underbrace{\gamma^{0}}_{\gamma^{0\dagger}=\gamma^0}\psi = \bar{\psi}\psi \text{ invariant (scalar)}$$

on the other hand

$$\bar{\psi}\gamma^5\psi \to \psi^{\dagger}\cancelto{I}{\gamma^{0\dagger}\gamma^0}\gamma^5\gamma^0\psi = \psi^{\dagger}\gamma^5\gamma^0\psi = -\psi^{\dagger}\gamma^0\gamma^5\psi = -\bar{\psi}\gamma^5\psi \text{ (pseudo-scalar)}$$

Similarly $\bar{\psi}\gamma^\mu\psi \to -\bar{\psi}\gamma^\mu\psi$ (vector)

$\bar{\psi}\gamma^\mu\gamma^5\psi \to \bar{\psi}\gamma^\mu\gamma^5\psi$ (pseudovector or axial-vector)

Now what is interesting is that when we for spinor sandwiches w/ weak interaction vertices we get:

$$\bar{\psi}\gamma^\mu(1-\gamma^5)\psi = \bar{\psi}\gamma^\mu\psi - \bar{\psi}\gamma^\mu\gamma^5\psi = \text{Vector} - \text{Axial Vector} \quad (\text{hence } c^V, c^A \text{ labels})$$

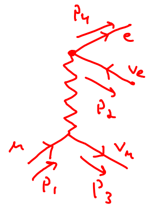
But such a difference breaks parity! Note, an overall + or − is fine, but here we have a difference w/ two different signs, so for example $\underbrace{2-5}_{-3} \to \underbrace{-2-5}_{-7}$

But we already knew the weak interactions break parity since they only ever produce left-handed neutrinos.

Monday, April 14, 2014
7:52 AM

One of the important things about the Weak interactions ($W^{\pm}$ to be specific) is that they provide the mechanism for true particle decay (not annihilation). This leads to useful lifetime results.

<u>Muon decay</u> $\mu \rightarrow \nu_\mu + e + \bar{\nu}_e$



$$\Rightarrow \quad \mathcal{M} = \bar{u}(3)\frac{-ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)u(1)\underbrace{\frac{-i(g_{\mu\nu}-q_\mu q_\nu/m_w^2c^2)}{q^2-m_w^2c^2}}_{\substack{\text{If } q \ll m_w c \text{ (typical for low-energy muons)} \\ \approx \frac{ig_{\mu\nu}}{m_w^2c^2}}}\bar{u}(4)\frac{-ig_w}{2\sqrt{2}}\gamma^\nu(1-\gamma^5)v(2)$$

$$\mathcal{M} \approx \frac{g_w^2}{8m_w^2c^2}\bar{u}(3)\gamma^\mu(1-\gamma^5)u(1)\bar{u}(4)\gamma_\nu(1-\gamma^5)v(2)$$

As usual we average over incoming spins and sum over final spins using Casimir's trick and the γ matrix trace theorems:

$$\langle|\mathcal{M}|^2\rangle_1 = 2\left(\frac{g_w}{m_w c}\right)^4(p_1\cdot p_2)(p_3\cdot p_4)$$

To evaluate the muon lifetime as seen in its rest frame we use $p_1 = (m_\mu c, \vec{0})$ to obtain (after some work):

$$\langle|\mathcal{M}|^2\rangle_1 = \left(\frac{g_w}{m_w c}\right)^4 m_\mu^2 E_2(m_\mu c^2 - 2E_2)$$

This can be used in Fermi's Golden Rule for decays (the full integral form, not the simple 2-body decay form!) and working w/ $m_e \approx 0$ since $m_e c^2$ is a small percentage of the energy released $(m_\mu - m_e)c^2$.
$\quad 105\ \frac{\text{MeV}}{c^2} \quad 0.5\ \frac{\text{MeV}}{c^2}$

$$d\Gamma = \frac{\langle|\mathcal{M}|^2\rangle_1}{2\hbar m_\mu}\frac{d^3p_2}{(2\pi)^3 2|\vec{p}_2|}\frac{d^3\vec{p}_3}{(2\pi)^3 2|\vec{p}_3|}\frac{d^3p_4}{(2\pi)^3 2|\vec{p}_4|}(2\pi)^4\delta^4(p_1-p_2-p_3-p_4)$$

The δ function can be used to evaluate $\int d^3\vec{p}_3$ and the remaining two momenta are bound to satisfy:

$|\vec{p}_2| < \frac{1}{2}m_\mu c$ max when $2 \longleftarrow \ \begin{smallmatrix}\longrightarrow 3 \\ \longrightarrow 4\end{smallmatrix}$ (decreases as)

$|\vec{p}_4| < \frac{1}{2}m_\mu c$ max when $4 \longleftarrow \ \begin{smallmatrix}\longrightarrow 2 \\ \longrightarrow 3\end{smallmatrix}$

$|\vec{p}_2| + |\vec{p}_4| > \frac{1}{2}m_\mu c$ min when $3 \longleftarrow \ \begin{smallmatrix}\longrightarrow 2 \\ \longrightarrow 4\end{smallmatrix}$ (increases as $3 \longleftarrow \begin{smallmatrix}\nearrow 2 \\ \searrow 4\end{smallmatrix}$)

After more work: $\tau = \frac{1}{\Gamma} = \left(\frac{m_w}{m_\mu g_w}\right)^4\frac{12\hbar(8\pi)^3}{m_\mu c^2}$

Using the observed muon lifetime 2.1970×10^{-6} s and $m_w = 80{,}420\ \frac{\text{MeV}}{c^2}$ $\Rightarrow g_w = 0.653 \Rightarrow \alpha_w = \frac{g_w^2}{4\pi} = \frac{1}{29.5}$

Compare to $\alpha = \frac{1}{137}$ for QED!
The weak force isn't <u>weak</u>, it's just that $W^{\pm}$ are so heavy!

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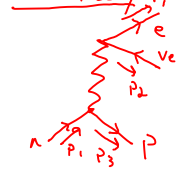
Neutron Decay p. $n \to p + e + \bar{\nu}_e$



$$\Rightarrow \langle |M|^2 \rangle_1 = 2 \left(\frac{g_w}{M_W c}\right)^4 (p_1 \cdot p_2)(p_3 \cdot p_4)$$ Note: Same expression as in muon decay!

This time when employing the Golden Rule, we cannot ignore the contribution from m_e to the energy released $(m_n - m_p - m_e)c^2$. This complicates the calculation considerably!

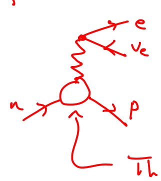
m_n: $939.6\ \frac{MeV}{c^2}$, m_p: $938.3\ \frac{MeV}{c^2}$, m_e: $0.5\ \frac{MeV}{c^2}$

After much work and some approximations: $\tau = \frac{1}{\Gamma} \simeq 1318\ s$ (using g_w found from muon decay)

But experimentally $\tau = 885.7\ \frac{MeV}{c^2}$

To be fair, it is hard to say just how W couples to the n or p since they are a "hot mess" (QCD) of quarks and virtual gluons and virtual quark-anti-quark pairs! This is not an issue when γ couples to the hot QCD mess since even in a mess of virtual pairs, the net electric charge is conserved. There is no theoretical starting point for "weak charge" conservation. So a similar calculation w/ QED gets a much better answer.

To quantify what is happening we say



This modified weak vertex can be written w/ $(1-\gamma^5) \to (c_V - c_A\gamma^5)$

Partially conserved axial current

Experimentally $c_V = 1$, $c_A = 1.27$ (determined by the same decay but within nuclei, e.g. $^{14}O \to {}^{14}N$).

Weak vector charge conservation (CVC hypothesis $\Rightarrow$ some symmetry protecting it)
conserved vector current
Somehow all the craziness inside the n+p does not change this part!

With this correction $\tau = 901 s$ with other corrections expected to bring it to the measured value.

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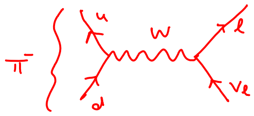
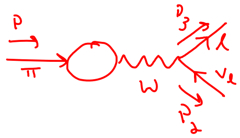
Pion decay $\pi^- \to \ell + \nu_\ell$



Actually more of a "scattering" process but the answer is interesting.

To calculate the actual π^- lifetime is challenging since it actually depends on the ground state wavefunction of the $d\bar{u}$ system. However what can be independently calculated is the ratio of $\pi^- \to e + \bar{\nu}_e$ to $\pi^- \to \mu + \bar{\nu}_\mu$.

To do this we use



$$\Rightarrow \mathcal{M} = \frac{g_W^2}{8(M_W c)^2}\, \bar{u}(3)\gamma_\mu(1-\gamma^5)v(2)\, F^\mu$$

encodes ignorance of ◯

Now F^μ must depend on some initial 4-vector, but the only one is $p^\mu \Rightarrow F^\mu = f_\pi p^\mu$ (scalar)
But f_π must depend on the only scalar available $p^2 = m_\pi^2 c^2$. This means that the result for F^μ is independent of which $\ell, \bar{\nu}_\ell$ we use!

Then $\langle|\mathcal{M}|^2\rangle = \frac{1}{8}\left[f_\pi\left(\frac{g_W}{M_W c}\right)^2\right]^2 \left[2(p\cdot p_2)(p\cdot p_3) - p^2(p_2\cdot p_3)\right]$

This time we can use the 2-body decay formula:

$$\Gamma = \frac{|\vec{p}_2|}{8\pi\hbar m_\pi^2 c}\langle|\mathcal{M}|^2\rangle$$

Since $|\vec{p}_2| = \frac{c}{2m_\pi}(m_\pi^2 - m_\ell^2)$

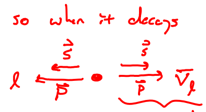
$$\Gamma = \frac{f_\pi^2}{\pi\hbar m_\pi^3}\left(\frac{g_W}{4M_W}\right)^4 m_\ell^2(m_\pi^2 - m_\ell^2)^2$$

We can't evaluate Γ w/out f_π, but we can evaluate:

$$\frac{\Gamma(\pi^- \to e^- + \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- + \bar{\nu}_\mu)} = \frac{m_e^2(m_\pi^2 - m_e^2)^2}{m_\mu^2(m_\pi^2 - m_\mu^2)^2} = 1.283 \times 10^{-4}$$

This should be surprising since we normally think that the kinematic likelihood is driven by mass differences and hence $\pi \to e + \bar{\nu}_e$ should be more likely than $\pi \to \mu + \bar{\nu}_\mu$.

What is going on? Well π^- is spin-0 so when it decays ℓ and ν_ℓ must come out w/ equal helicities (or opposite spins).



required since anti-neutrinos are always right-handed

But m_e is "almost" zero so it is almost always produced as a left-handed only particle whereas m_μ is very massive and so happy to exist as a right-handed particle.