

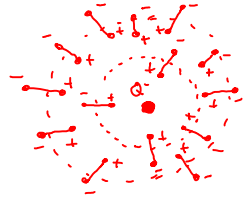
A Classical Fairy Tale

Consider a massive particle with unknown charge happily at rest: $\frac{e}{4\pi\epsilon} \dots \rightarrow \bullet Q^-$

If we wanted to measure the charge, we might scatter a known charge with known p^h and use the observed trajectory to infer q_m . If we used an electron, we could even work in the limit that the particle remains at rest the whole time.

If we knew the impact parameter b we could use: $Q^- = \frac{b \Delta E}{q_0 \cot(\frac{\theta}{2})}$

Now suppose the particle is submerged in a dielectric. Then the negatively charged particle will gently reorient the dipoles creating:



The "effective charge" due to dipole screening. For the inner Gaussian surface we have $Q + 4\epsilon$, while for the outer we have $Q + 9\epsilon$. Large distances \Rightarrow more screening.

Now the issue is that if we send in a probe electron, it will only respond to the effective charge which varies w/ distance. If we call the distance of closest approach \tilde{r} , then the largest (screened) value seen would be $Q(\tilde{r})$. But how close the electron comes (\tilde{r}) is determined by its momentum (p) so this could be rewritten as $Q_m(p)$.

So in this classical example we see a use for effective charges which vary w/ the momentum scale of the experiment. Of course we can always seek the classical "true" q charge by either probing w/ p high enough that we get closer to q than any screening plane, or simpler still we just remove q from the dielectric.

A Quantum Fairy Tale

Now suppose we have a Q^- in free space and we want to probe it w/ an e^- . We know that roughly:



and we replace definite results w/ probabilities.

However we know that we could also have:  and the e^+e^- pair could act like a dielectric.

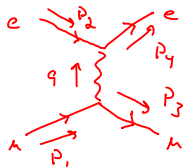
This turns out to be a crucial part of the story of QFT and particle physics, and will lead to the process of renormalization.

Renormalization

We want to explore the "screening" of real electric charge by virtual particle pairs in a more quantitative way. This will also lead us to understand how to handle many divergences encountered in Feynman amplitudes.

Consider $e^- + \mu^- \rightarrow e^- + \mu^-$

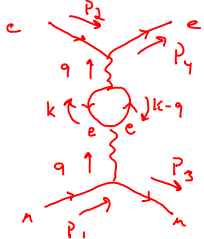
At 2nd order we have the "tree-level" contribution (simplest nontrivial):



$$\Rightarrow M_{tree} = -g_e^2 [\bar{u}(3)\gamma^\mu u(1)] \frac{\pi_{\mu\nu}}{q^2} [\bar{u}(4)\gamma^\nu u(2)]$$

$q \equiv p_2 - p_4$

The effects of virtual particle pairs start at 4th order with the most important contribution being:



We will later learn why the other 4th order contributions do not play a role in what is to come.
To evaluate a diagram like this with a purely internal loop of matter we need the Feynman rule: For internal loops of matter write down the ordered product of vertex factors and propagators, then take the trace (in spin space) and $\times (-1)$

$$\Rightarrow M_{1-loop} = -g_e^2 [\bar{u}(3)\gamma^\mu u(1)] i g_e^2 \underbrace{\int \frac{d^4k}{(2\pi)^4} \frac{\text{Tr}[\gamma_\mu(k+\hbar c)\gamma_\nu(k-q+\hbar c)]}{q^2(k^2-\hbar^2 c^2)[(k-q)^2-\hbar^2 c^2]}}_{\substack{\text{naively expect } \int \frac{k^3 dk k^3}{k^4} = \int k dk = k^2 |^\infty \rightarrow \infty \\ \text{but actually only } \ln k |^\infty}} [\bar{u}(4)\gamma^\nu u(2)]$$

Call this $-i g_e^2 \mathcal{I}(q^2)$
due to $\text{Tr}(\gamma_\mu \gamma_\nu) = 4\eta_{\mu\nu}$

hell of a lot of

After some massaging: $\mathcal{I}(q^2) = -\frac{1}{i2\pi^2} \left\{ \underbrace{\int_{\hbar c}^{\infty} \frac{dz}{z}}_{\substack{\text{blows up} \\ \text{as } \ln z}} - 6 \int_0^1 z(1-z) \ln \left[1 - \frac{q^2}{\hbar^2 c^2} z(1-z) \right] dz \right\}$ $q^2 < 0$ for this process!

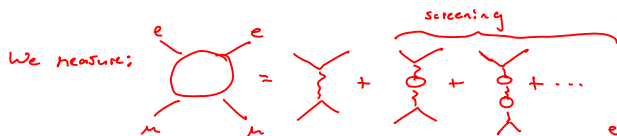
finite $f\left(\frac{-q^2}{\hbar^2 c^2}\right)$ w/ $f(-q^2=0) = 0$ and $f(q^2)$ increasing w/ $|q^2|$

The next step in the "renormalization" program is to "regularize" the divergence (make it a finite contribution). We will simply use an upper "cutoff": $\int_{\hbar c}^{\hbar c} \frac{dz}{z} = \ln\left(\frac{M_c^2}{\hbar^2 c^2}\right)$

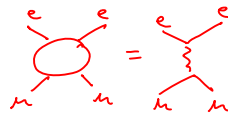
Then: $\mathcal{I}(q^2) = -\frac{1}{i2\pi^2} \left\{ \ln \frac{M_c^2}{\hbar^2 c^2} - f\left(\frac{q^2}{\hbar^2 c^2}\right) \right\}$ notice that divergence is still present when $\hbar c \rightarrow \infty$

And: $M_{tree} + M_{1-loop} = -g_e^2 [\bar{u}(3)\gamma^\mu u(1)] \frac{\pi_{\mu\nu}}{q^2} \left(1 - \frac{g_e^2}{i2\pi^2} \left\{ \ln \frac{M_c^2}{\hbar^2 c^2} - f\left(\frac{q^2}{\hbar^2 c^2}\right) \right\} \right) [\bar{u}(4)\gamma^\nu u(2)]$

So how do we interpret this?



But interpret our measurement "classically", i.e.



so we really measure an "effective" charge that includes all loop contributions

To formalize this we simply rewrite our results in terms of the effective or "renormalized" charge/coupling:

$$g_R(q^2) \equiv g_e \sqrt{1 - \frac{g_e^2}{12\pi^2} \left[\ln\left(\frac{M_0^2}{\mu^2}\right) - f\left(\frac{-q^2}{\mu^2}\right) \right] + \dots} = \text{physically measured value of } e\sqrt{\frac{4\pi}{k\epsilon}} \text{ in experiment at momentum transfer scale } q = |p_2 - p_4|$$

$$\text{Then of course: } M_{tree} + M_{1-loop} = -g_R(q^2) [\bar{u}(3)\gamma^\mu u(1)] \frac{j_{\mu\nu}}{q^2} [\bar{u}(4)\gamma^\nu u(2)]$$

which looks like it came from alone!

This is an example of an "effective" theory where the quantum loop corrections are bundled into renormalized quantities and the result is interpreted as the "classical" tree-level, i.e.



This is often written in terms of the zero-momentum (large distance) value: $g_R(0) = g_e \sqrt{1 - \frac{g_e^2}{12\pi^2} \ln\left(\frac{M_0^2}{\mu^2}\right)}$ to obtain:

$$g_R(q^2) = g_R(0) \sqrt{1 + \frac{g_R(0)}{12\pi^2} f\left(\frac{-q^2}{\mu^2}\right)}$$

So lets compare this to the screening story. If we are very far away ($q^2 \rightarrow 0$) $\Rightarrow g_R(q^2) = g_R(0)$. As we probe closer, i.e. $|q^2| > 0$ and since $f\left(\frac{-q^2}{\mu^2}\right)$ increases we find that: $g_R(q^2 \neq 0) > g_R(0)$, i.e. we had screening!

Okay fine, but what about the divergent $\ln\left(\frac{M_0^2}{\mu^2}\right)$ term in $g_R(0)$? Isn't it causing everything to blow up? Well it depends on your input. We built our fundamental theory w/ a coupling $g_e = e\sqrt{\frac{4\pi}{k\epsilon}}$ but what is the fundamental (or "bare" or "unscreened") value of e ? Unlike in the classical example, we cannot simply remove the μ from this "selective bath". Worse still, to use higher $|q^2|$ to probe beyond any screening actually requires $|q^2| \rightarrow \infty$ which we simply cannot do!

Are we lost? No, not really. After all, we can (and do) measure $g_R(0)$ itself (in fact that is the value you see in tables), and then we can just work in terms of $g_R(0)$ instead of the unknowable g_e . The upshot is that working in terms of $g_R(0)$ means the problematic infinities no longer appear!

Three important points:

1. In principle this should be done to all orders (# of loops) to get the correct $g_R(q^2)$, but lower # dominates.
2. We encountered ∞ 's, but now realize that this was due to erroneously using the measured electron charge where we should be using the, unknown, bare value. This situation crops up in many QFTs, and when we can "fix" it like this, we call the theory renormalizable. The standard model is, but perturbative quantum gravity is not.
3. Even without the problem of ∞ 's, we see the need for renormalizing because the truth is that measured values of "constant" actually depend on q , i.e. they run (running couplings).

Often people will leave out the q^2 dependence and work in terms of $g_R(0)$. But then they must include all loop corrections. But these will now be finite since they are in terms of $g_R(0)$ and not g_e !