Review

a) Wavefunction normalized $\int_{-\infty}^{\infty} \Psi^* \Psi \, dx = 1$ preserved by the TDSE.

b) Expectation values in QM: $\langle f \rangle = \int_{-\infty}^{\infty} \Psi^* f \Psi \, dx$

$$\langle p \rangle = \int_{-\infty}^{\infty} \Psi^* \left(-i\hbar \frac{\partial}{\partial x}\right) \Psi \, dx$$

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

c) Uncertainty princ. For any solution to TDSE

$$\underbrace{\sigma_x \sigma_p \geq \frac{\hbar}{2}}_{\text{will prove later}}$$

also holds for any pair of conjugate

We $\langle x \rangle, \langle p \rangle$ or $\hat{x} = x$ $\hat{p} = -i\hbar \frac{\partial}{\partial x}$

$$T(x,p) = \frac{p^2}{2m} \Rightarrow \hat{T} = \frac{\left(-i\hbar \frac{\partial}{\partial x}\right)^2}{2m}$$

in general

$$Q(x,p) \Rightarrow \hat{Q}(\hat{x}, \hat{p}) \qquad Q = xp \overset{?}{\Rightarrow} \hat{x}\hat{p} = x\left(-i\hbar \frac{\partial}{\partial x}\right)$$

$$Q = px \Rightarrow \hat{p}\hat{x} = -i\hbar \frac{\partial}{\partial x} x$$

Solving Schrödinger

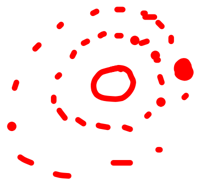
TDSE : $i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi$

potential energy func.

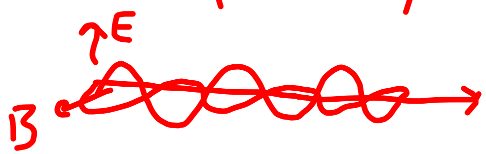
In gen. $V(x,t)$

But we will assume $V(x)$

Hydrogen Atom: $V = -\frac{K}{r}$



Transitions : Incorporate light



$\vec{E}(t) = \hat{\epsilon} E \cos(\omega t)$

$V(x,t) = -qE_0 x \cos(\omega t)$

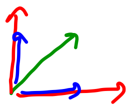
very hard chapter 9 crap!

T.D.P.T.

Functions as vectors:

$$\text{Taylor: } f(x) = f(0) + f'(0)x + \frac{1}{2!}f''(0)x^2 + \cdots$$
$$= \sum_{n=0}^{\infty} \frac{1}{n!} f^{n}(0)\, \underline{\underline{x^n}}$$

$$\text{Fourier: } f(x) = \sum_{n=-\infty}^{\infty} c_n \underbrace{e^{inx}}_{\text{sines + cosines}}$$



Solutions to the TDSE $\Psi(x,t)$ w/ $V(x)$

can be decomposed: $\underbrace{\Psi(x,t)}_{\text{general solution}} = \sum_{n=1}^{\infty} c_n \underbrace{\Psi_n(x,t)}_{\text{basis functions are seperable}}$

Seperable solution: $\Psi_n(x,t) = \phi_n(t)\,\psi_n(x)$

Example: $\underbrace{\Psi(x,t)}_{\text{Is this seperable?}} = \underbrace{\frac{1}{\sqrt{2}}\phi_1(t)\psi_1(x) + \frac{1}{\sqrt{2}}\phi_2(t)\psi_2(x)}_{\stackrel{?}{=} f(t)\, g(x)}$

Next time: How do we find $\phi_n(t)$, $\psi_n(x)$?

Then $\Psi = \sum_{n=1}^{\infty} \underline{\underline{c_n}}\, \underline{\underline{\phi_n(t)\psi_n(x)}}$

$$\psi(x,t) = A e^{-\frac{x^2}{a}} e^{-i\omega t}$$

$$\int_{-\infty}^{\infty} \psi^* \psi \, dx = 1$$

$$A^2 \int_{-\infty}^{\infty} e^{-\frac{x^2}{a}} \cancel{e^{i\omega t}} e^{-\frac{x^2}{a}} \cancel{e^{-i\omega t}} \, dx = 1$$

$$\frac{d}{dt} \int_{-\infty}^{\infty} \psi^* \psi \, dx = 0$$

$\Rightarrow$ you can use any time you want

so why not $t=0$!