Solving Schrödinger

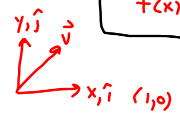
$\Psi(x,t) = \sum_n c_n \underbrace{\Psi_n(x,t)}$

separable $\Rightarrow \Psi_n(x,t) = \phi_n(t)\psi_n(x)$

just a number given

Example : $\frac{d^2 f(x)}{dx^2} = -k^2 f(x) \Rightarrow f(x) = \sin(kx), \cos(kx)$

(0,1)



(1,0)

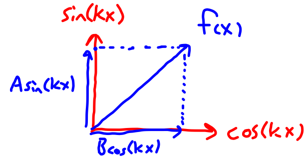
$\hat{i}\cdot\hat{j} = 0 = (0\ 1)\binom{1}{0} = 0$

$\hat{i}\cdot\hat{i} = 1 = (0\ 1)\binom{0}{1} = 1$

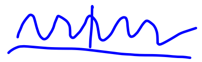
$\hat{j}\cdot\hat{j} = 1 = (1\ 0)\binom{1}{0} = 1$

$f(x) = A\sin(kx) + B\cos(kx)$

$$\int_{-\infty}^{\infty} \sin(kx)\cos(kx)\,dx = 0$$



Non normalizable $\longrightarrow \int_{-\infty}^{\infty} \cos^2(kx)\,dx = \infty$



$f(x) = Af_1(x) + Bf_2(x)$

ortho $\int_{-\infty}^{\infty} f_1(x) f_2(x)\,dx = 0$

norm $\int_{-\infty}^{\infty} f_i^2(x)\,dx = 1$

$$A = f(x)\cdot f_1(x) = \int_{-\infty}^{\infty} (Af_1(x) + Bf_2(x)) f_1(x)\,dx$$

$$= \int_{-\infty}^{\infty} A \cancel{f_1^2 dx}^{\,1} + \int_{-\infty}^{\infty} B \cancel{f_2 f_1}^{\,0} dx$$

$$= A$$

Example

$f(x) = A\sin(kx)$
$+ B\cos(kx)$

$\boxed{\begin{array}{l} f(0) = \frac{1}{2} \\ f(\frac{\pi}{2k}) = 3 \end{array}}$ $\begin{array}{l} = B \\ = A \end{array}$

b.c.s

Example

$f(0) = \frac{1}{2}$

$f'(0) = 1$

T.D.S.E. $\quad i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V(x)\Psi$

$$\Psi(x,t) = \sum_n c_n \underbrace{\psi_n(x,t)}_{\text{seperable}}$$

Seperable : a) stationary

b) have definite energy

Assume $\psi_n(x,t) = \phi_n(t)\psi_n(x) \Rightarrow \frac{\partial \psi_n}{\partial t} = \frac{\partial \phi_n}{\partial t}\psi_n$

Method sep. of variables : $\quad \frac{\partial^2 \psi_n}{\partial x^2} = \phi_n \frac{\partial^2 \psi_n}{\partial x^2}$

$$\underbrace{\frac{i\hbar \frac{\partial \phi_n}{\partial t}\cancel{\psi_n}}{\phi_n \cancel{\psi_n}}}_{\text{only functions of } t} = \underbrace{\frac{-\frac{\hbar^2}{2m}\cancel{\phi_n}\frac{\partial^2 \psi_n}{\partial x^2}}{\cancel{\phi_n}\psi_n} + \frac{V\cancel{\phi_n}\cancel{\psi_n}}{\cancel{\phi_n}\cancel{\psi_n}}}_{\text{only depends on } X}$$

crucial that $V(X)$

a) $\frac{i\hbar \frac{\partial \phi_n}{\partial t}}{\phi_n} = E_n = \text{constant}$ [we can solve now!]

b) $\boxed{\frac{-\frac{\hbar^2}{2m}\frac{\partial^2 \psi_n}{\partial x^2}}{\psi_n} + V = E_n}$ [we cannot solve what $V(x)$]

a) $\frac{\partial \phi_n}{\partial t} = \frac{E_n}{i\hbar}\phi_n \Rightarrow \phi_n = Ae^{-i\frac{E_n}{\hbar}t}$

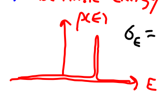
A: assume $= 1$

$$\underbrace{\phi_n(t) = e^{-i\frac{E_n}{\hbar}t}}_{\text{Stationary seperable solutions}}$$

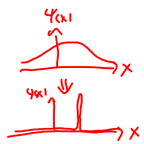
Stationary - expectation values do not depend on time

$$\langle f \rangle = \int_{-\infty}^{\infty} \psi_n^* \phi_n^* f \phi_n \psi_n \, dx = \int_{-\infty}^{\infty} \psi_n^* f \psi_n \, dx$$

b) Definite energy



$\sigma_E = 0$



$$\underbrace{-\frac{\hbar^2}{2m}\frac{\partial^2 \psi_n}{\partial x^2} + V\psi_n = E_n\psi_n}$$ T.I.S.E.

$$\underbrace{\left[\frac{\left(-i\hbar\frac{\partial}{\partial x}\right)^2}{2m} + V\right]}\psi_n = E_n\psi_n$$

$$\left[\underbrace{\frac{p^2}{2m}} + V(x)\right]\psi_n = E_n\psi_n$$

Hamiltonian $= \hat{H}(\hat{p}, \hat{x})$

TISE

$\hat{H}\psi_n = E_n\psi_n$

$\sigma_E = \sqrt{\langle H^2\rangle - \langle H\rangle^2} = 0 \qquad \langle H \rangle = \int_{-\infty}^{\infty} \psi_n^* \underbrace{\hat{H}\psi_n}_{E_n\psi_n \text{ TISE}} dx =$

$\langle H^2 \rangle = E_n^2 \quad \langle H \rangle^2 = E_n^2 \qquad = E_n \int_{-\infty}^{\infty} \psi^*\psi \, dx = E_n$