Review:

must always be normalized; must be normalized

Discrete: $\Psi(x,t) = \sum_n c_n \psi_n(x,t)$ $\quad c_n = \int_{-\infty}^{\infty} \psi_n^*(x,0)\,\Psi(x,0)\,dx$

$\psi_n(x,t) = \phi_n(t)\,\psi_n(x)$

$\phi_n(t) = e^{-i\frac{E_n}{\hbar}t}$ $\quad \psi_n(x)$: TISE (V(x))

must be normalized; do not have to be normalizable

Continuous: $\Psi(x,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\phi(k)\,\psi(k,x,t)\,dk$ $\quad \phi(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\psi^*(k,x,0)\,\Psi(x,0)\,dx$

free particle: $e^{-i\frac{\hbar k^2}{2m}t}\,e^{ikx}$

(Bound states)

Discrete - arise when motion is bounded (can not visit x=+∞ or -∞)

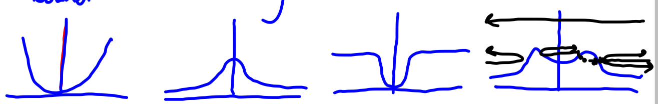
Continuous - arise when motion is unbounded (on at least one side) (can visit x=+∞ or x=-∞)

(Scattering States)

Can we get both?

	Bound	Scattering	S+B States	S+B states
Classical	(includes ∞-well)	(includes V=0)		

	Bound	Scattering	S+B states	Only S states
Quantum				



To determine the states in classical requires detailed info.

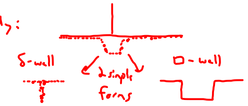
In QM all we need is how E_{tot} compares $V(\pm\infty)$.

$E_{tot} < V(\pm\infty)$ $\quad E_{tot} > V(\pm\infty)$ $\quad E_{tot} > V(\pm\infty)$ S, $E_{tot} < V(\pm\infty)$ B $\quad E_{tot} > V(\pm\infty)$ S

We can always choose $V(\pm\infty) = 0$!

$E_{tot} < 0$ B $\quad E_{tot} > 0$ S $\quad E_{tot} > 0$ S, $E_{tot} < 0$ B $\quad E_{tot} > 0$ S

Now study:



<u>General Approach</u>

Solve TISE in each regions.

Use constraints to patch solutions.

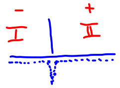
TISE $\underbrace{-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi = E\psi}_{\text{3 pieces of information}}$

∞-well

$\psi_I = 0$ $\psi_{III} = 0$

ψ_{II} used continuity @ $x = \pm a$ + normalization

<u>δ-function well</u>



$V(x) = -\alpha\,\delta(x)$ $\delta(x) = \begin{cases} \infty & x = 0 \\ 0 & x \neq 0 \end{cases}$

∞ deep, ∞ narrow } conspire to give finite answers

$\int_{-\infty}^{\infty} \delta(x)\,dx = 1$

TISE: $-\frac{\hbar^2}{2m}\frac{d^2\psi_n}{dx^2} - \alpha\,\delta(x)\,\psi_n = E_n\psi_n$

Since $V(\pm\infty) = 0$ we know $\begin{cases} E_{tot} > 0 & \text{S states} \\ E_{tot} < 0 & \text{B states} \end{cases}$

Today Bound States ($E_n < 0$)

$x > 0$ $-\frac{\hbar^2}{2m}\frac{d^2\psi_n^+}{dx^2} = E_n\psi_n^+ \Rightarrow \frac{d^2\psi_n^+}{dx^2} = \underbrace{-\frac{2mE_n}{\hbar^2}}_{>0}\psi_n^+$

$\frac{d^2\psi_n^+}{dx^2} = k^2\psi_n^+$

$\psi_n^+ = \underbrace{A_+e^{kx}}_{\to\infty \text{ @ } x\to+\infty} + B_+e^{-kx}$

$\Rightarrow A_+ = 0$

$x < 0$ $\psi_n^- = A_-e^{kx} + \underbrace{B_-e^{-kx}}_{\to\infty \text{ @ } x\to-\infty}$

$B_- = 0$

So far we don't know k, B_+, A_-

Need more constraints:

Continuity: $\psi^+(x\to 0_+) = \psi^-(x\to 0_-)$

$B_+ = A_- = A \Rightarrow \boxed{\psi(x) = Ae^{-k|x|}}$

Still don't know A, k

Could normalize to find A (next time!)