CM: Solve e.o.m. $\Rightarrow \vec{x}(t)$
QM: Solve e.o.m. $\Rightarrow \psi(x,t)$

<u>State</u> - determines outcome of measurement
$\vec{x}(t)$ gives position at time $t \Rightarrow \vec{p}, E$
$\psi(x,t)$ gives probability of x at $t \Rightarrow$ expectation values of $\vec{p}, E$

$$\psi(x,t) = a_1 \psi_1 + a_2 \psi_2 + \cdots$$

prob. of given outcome

If $a_n \neq 0$ for more than one $n \Rightarrow$ superposition state
otherwise pure

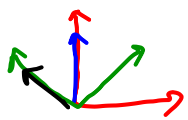
Measurements return pure states

$$\psi(x,t) = 0 + \cdots + \psi_n + \cdots + 0$$

Expansion basis is determined by the measurement made.

Measure E: $\psi(x,t) = \sum_n a_n \psi_n^H$

After measurement of E: $\psi(x,t) = a_n \psi_n^H$



collapse of wavefunction

$\psi(x,t) \Rightarrow$ time evolution is governed by the S.E.

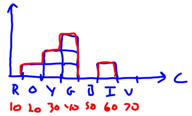
Probability

collection of possible outcomes → discrete (quantized observables) / continuous (cont. observables)

Example:

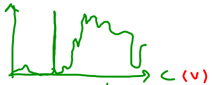
collection of colored balls

$N(c)$ Discrete



$N(v)$ Continuous

$N(c)$



number density (# per color)

$N = \int_c \underbrace{N(c)}\, dc$

$P(c) = \frac{N(c)}{N} = \frac{N(c)}{\int_c N(c)\,dc}$

Most probable: max $P(c)$

$P(Y) = \frac{2}{7}$

$P(G) = \frac{3}{7}$

$P(R) = 0 = \frac{0}{7}$

in general

$P(c) = \frac{N(c)}{N}$ ← # of balls

Normalization constant

$N = \sum_c N(c) = \sum_c \underbrace{N(c)}_{\text{number density}} \cdot \overset{1}{\overset{\shortparallel}{A}} = \sum_c N(c) \cdot 1 \times \text{width}$

Most probable: Green (more green balls) ⇒ max. $P(c)$

What is prob. of getting Y or G?

$P(Y \text{ or } G) = \frac{5}{7} = \frac{2}{7} + \frac{3}{7}$

$= P(Y) + P(G)$

Prob. over a range: $\int_a^b P(c)\,dc$

What is prob. of getting any color?

$\sum_c P(c) = 1 = \frac{\sum_c N(c)}{N} = \frac{N}{N}$

$P(c) = \frac{N(c)}{N}$

$\int_c P(c)\,dc = 1$

What is the median or average?

median: need ordering } Numbers

average: combine outcomes }

median $20 + 30 + 30 + 40 + 40 + 40 + 60$

Median v is s.t.

$\int_0^v P(v)\,dv = \frac{1}{2}$

or

$\int_v^\infty P(v)\,dv = \frac{1}{2}$

Average: $\frac{20+30+30+40+40+40+60}{7} = 37.14$

$20P(20) + 30P(30) + 40P(40) + 60P(60) = 37.14$

$\sum_v v P(v)$ → Average of v: $\langle v \rangle = \int_0^\infty v P(v)\,dv$

Average v^2 $\frac{20^2+30^2+\dots}{7} = 151.42$

$\sum_v v^2 P(v)$ → Average of v^2: $\langle v^2 \rangle = \int_0^\infty v^2 P(v)\,dv$

$\sum_v f(v) P(v)$ → $\langle f(v) \rangle = \int_0^\infty f(v) P(v)\,dv$