Review: TDSE $i\hbar \frac{\partial \Psi}{\partial t} = \hat{H}\Psi$ $\qquad \int \Psi^* \Psi \, dx = 1$

Assume: $\Psi = \begin{cases} \sum_n c_n \psi_n(x,t) \\ \frac{1}{\sqrt{2\pi}} \int \phi(k)\, \psi(k,x,t)\, dk \end{cases}$

$\langle \hat{f} \rangle_{st} = \int \psi_n^* \hat{f} \psi_n \, dx$

stationary states

a) $\frac{d\langle \hat{f} \rangle_{st}}{dt} = 0$

b) $\underbrace{\hat{H}\psi_{st} = E\psi_{st}}_{\text{TISE}}$

$c_n = \int_{-\infty}^{\infty} \psi_n^* \Psi \, dx$

$\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi^*(k)\, \Psi \, dx$

$\vec{v} \Rightarrow v_x = \hat{i} \cdot \vec{v}$

Vector space: a) scalar multiplication ($\mathbb{R}$ or $\mathbb{C}$) $c\vec{v} = \vec{w}$

b) mult. is distributive & associative

c) vector addition $\vec{v} + \vec{w} = \vec{u}$

d) add is commutative & associative

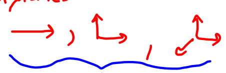
e) zero vector $\vec{v} + \vec{0} = \vec{v}$

f) inverse $\vec{v} + (-\vec{v}) = \vec{0}$

What is the vector space we are dealing with?

In CM: Dimension of dynamics determines the vector space.

$\vec{x}(t)$



you do <u>not</u> have to use a rectangular basis

In QM: Solutions to TDSE

Limited to "normalizable" functions.

$\Psi(x,t)$

We want $\boxed{\int_a^b \Psi^* \Psi \, dx < \infty}$ $\quad L_2(a,b)$ ← Lebesque

$\|x\| = (x_1^2 + x_2^2 + \dots)^{\frac{1}{2}}$

"Square-integrable functions"

Why not restrict to $\int \Psi^* \Psi \, dx = 1$? Vector addition is not closed

allowed

X

We have a vector space, now we need a way to represent them: basis

We would like:

requires a product between vectors

linearly independent set of orthonormal vectors that span $L_2(a,b)$

one basis vector cannot be represented by a linear comb. of others

$\hat{i} \neq a\hat{j} + b\hat{k}$

e.g. $\hat{i}\cdot\hat{j} = \delta_{ij}$

any function in $L_2(a,b)$ can be represented by a linear of basis vectors

requires only vector addition and scalar mult.

Vector multiplication

In CM: $\hat{i}\cdot\hat{j} \in \mathbb{R}$ $\Rightarrow$ $\vec{V}\cdot\vec{W} = (v_x\hat{i} + v_y\hat{j})\cdot(w_x\hat{i} + w_y\hat{j})$

$= V_x w_x \hat{i}\cdot\hat{i} + v_y w_y \hat{j}\cdot\hat{j}$
$+ V_x w_y \hat{i}\cdot\hat{j} + v_y w_x \hat{j}\cdot\hat{i}$

So we only need

In QM: $\underbrace{\int_a^b \psi_m^* \psi_n \, dx}_{\langle \psi_m | \; | \psi_n \rangle} \in \mathbb{C}$

$\langle \psi_m |$ bra, $| \psi_n \rangle$ ket

Orthonormality: $\langle \psi_n | \psi_m \rangle = \delta_{nm}$

An important result Schwarz inequality

$$\left| \int_a^b f^* g \, dx \right| \leq \sqrt{\int_a^b |f|^2 dx \int_a^b |g|^2 dx}$$

$$\Downarrow$$

$$\vec{V}\cdot\vec{W} \leq \sqrt{\vec{V}\cdot\vec{V} \;\; \vec{W}\cdot\vec{W}}$$

$$VW\cos\theta \leq VW$$

The vector space $L_2(a,b)$ combined with the inner product $\langle \; | \; \rangle$ is a Hilbert Space.

Title: Feb 11 - 11:52 AM (2 of 2)