Pick a $V(x)$

Find c_n if $\Psi(x,0) = f(x)$.

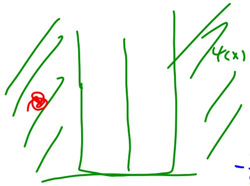
Check $\sigma_x \sigma_p \geq \frac{\hbar}{2}$ for $\psi_n(x,t)$

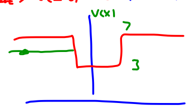
Expectation of $\psi_n(x)$:

Bound – (discrete) $E_{tot} < V(\pm\infty) \overset{conv.}{=} 0$ (normalizable)

Scattering – (cont. spectrum) $E_{tot} > V(\pm\infty) = 0$ (non-normalizable)

Consider $V(x = \pm\infty)$





TDSE

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi}$$

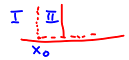
Used S.O.V. to S.O.L.V.E.

$$-\frac{i\hbar}{\phi}\frac{d\phi}{dt} = E \qquad E = -\frac{\hbar^2}{2m}\frac{1}{\psi}\frac{\partial^2\psi}{\partial x^2} + V$$

TISE : $$-\frac{\hbar^2}{2m}\frac{d^2\psi_n(x)}{dx^2} + V(x)\psi_n(x) = E_n\psi_n(x)$$

$\psi(k,x)$ for cont. ($E(k)$)

Break $V(x)$ into regions.



Join solutions:

a) Continuity of ψ: $\psi^{I}(x_0) = \psi^{II}(x_0)$

c) smoothness : $\left.\frac{d\psi^{I}}{dx}\right|_{x_0} = \left.\frac{d\psi^{II}}{dx}\right|_{x_0}$ but!!

(cannot use smoothness when $U(x)$ makes an ∞ jump!)

Solving TISE for $\psi_n(x)$ will involving finding E_n!!

$$\phi = e^{-\frac{iE_n}{\hbar}t}$$

Big bad azz $\Psi(x,t) = \sum_n c_n \phi_n(t)\psi_n(x)$

$$C_n(t_0) = \int_{-\infty}^{\infty}\psi_n^*(x)\phi_n^*(t_0)\Psi(x,t_0)\,dx$$

$\phi_n^* = e^{+\frac{iE_n}{\hbar}t}$ $$= \int_{-\infty}^{\infty}\psi_n^*(x)\phi_n^*(t_0)\left(\sum_m c_m\phi_m(t_0)\psi_m(x)\right)dx$$

$\phi_m = e^{\frac{iE_m}{\hbar}t}$ $$= \int_{-\infty}^{\infty}\sum c_m\phi_n^*\phi_m\,\psi_n^*\psi_m\,dx$$

$$= \sum_m c_m \cancel{\phi_n^*\phi_m}\underbrace{\int_{-\infty}^{\infty}\psi_n^*\psi_m\,dx}_{\delta_{nm}} \qquad \delta_{nm} = \begin{cases}0 & n\neq m\\ 1 & n=m\end{cases}$$

$$= C_n$$

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \phi(k)\, \underbrace{\phi(k,t)}_{e^{-\frac{iE(k)}{\hbar}t}}\, \underbrace{\psi(k,x)}_{\text{solu}}\, dk \quad \text{TISE}$$

$$\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi^*(k,x,t)\, \Psi(x,t)\, dx$$