Review: CM. e.o.m. + b.c. $\Rightarrow \vec{x}(t) \Rightarrow (E, \vec{p}, \text{etc.})$

QM. TDSE + b.c. $\Rightarrow \underbrace{\Psi(x,t)}_{} \Rightarrow (E_{avg}, p_{avg}, \text{etc}_{avg})$

$\Psi^*_{(x)} \Psi_{(x)}$ = prob. density for finding the particle at x

$$\Psi(x,t) = \int_a^b \Psi(y,t)\, \delta(y-x)\, dy$$

$\Phi(p,t)$

Ψ must be normalized $\Rightarrow$ must live in $L_2(a,b)$

+ inner product

Hilbert Space

What is $\Psi(x,t)$ and what do we do with it?

Momentum: If we had started w/ $\Phi(p,t) \Rightarrow \hat{p} f_p(x) = p f_p(x)$

$$\Psi(x,t) = \int \Phi(p,t) f_p(x)\, dp$$

For $\Psi(x,t)$: $\langle \hat{p} \rangle = \dfrac{d\langle \hat{x} \rangle}{dt} \Rightarrow \hat{p} = -i\hbar \dfrac{\partial}{\partial x}$

$$\sigma_{\hat{A}} = \sqrt{\langle \hat{A}^2 \rangle - \langle \hat{A} \rangle^2}$$

$\sigma_{\hat{x}} \sigma_{\hat{p}} \geq \dfrac{\hbar}{2} \Rightarrow$ G.U.P. $\sigma^2_{\hat{A}} \sigma^2_{\hat{B}} \geq \left[\frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle\right]^2$

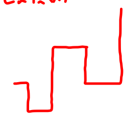
Getting Ψ? $i\hbar \dfrac{\partial \Psi(x,t)}{\partial t} = \hat{H}\, \Psi(x,t)$ $\quad \cancel{\hat{H}\Psi = E_n \Psi}$

$\Downarrow$?

$$\cancel{i\hbar \frac{\partial \Psi}{\partial t} = E_n \Psi(x,t)}$$

Assume: $\Psi(x,t) = \sum_n c_n \underbrace{\psi_n(x,t)}_{\phi_n(t)\, \psi_n(x)}$

Used sep. of var. on $\psi_n(x,t) \Rightarrow \begin{cases} \phi_n(t) = e^{-i\frac{E_n}{\hbar}t} \\ \hat{H}\psi_n(x) = E_n \psi_n(x) \quad \text{TISE} \end{cases}$



$\hat{H}(\hat{p}, \hat{x}) = \dfrac{\hat{p}^2}{2m} + V(\hat{x})$: $\begin{cases} V(\hat{x}) - \text{piecewise constant} \\ V(\hat{x}) - \text{harmonic oscillator (algebraic method)} \\ V(\hat{x}) - \delta\text{-function} \end{cases}$

exactly solvable

2 types of solutions: a) scattering (non-normalized)

b) bound (normalizable)

What next? 2 generalizations

i) 1D → 3D (new coordinates, angular momentum, hydrogen, degeneracy)

Degeneracy ⇒ many eigenfunctions w/ single Eigenvalue

Example: Free particle in 2D $\hat{H}\Psi = E\Psi$

Solution: add a label (p_x)

~~Could I use $\hat{H}, \hat{x}$~~

Must use compatible operators.

ii) 1 particle → many particles (bosons/fermions, stat. mech.)