Avg. 75.02

High 92.5

In stationary states $\frac{d\langle \hat{f} \rangle}{dt} = 0$ $\quad \phi_1^*(t) = e^{i\frac{E_1}{\hbar}t}$ $\quad \phi_2^*(t) = e^{i\frac{E_2}{\hbar}t}$

$$\Psi = \sum_n c_n \psi_n \overset{e.g.}{=} c_1\psi_1 + c_2\psi_2$$

$$\langle \vec{x} \rangle = \int \Psi^* x \Psi \, dx = \int (c_1^*\psi_1^* + c_2^*\psi_2^*)x \, (c_1\psi_1 + c_2\psi_2)\, dx$$

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}$$

$$\sigma_x = \sqrt{\langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2}$$

$$\sigma_p = \sqrt{\langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2}$$

≥ 0

$\langle \hat{H} \rangle = 32\,J$ $\quad \langle \hat{H}^2 \rangle$ $\quad \sigma_H = \sqrt{\underbrace{\langle \hat{H}^2 \rangle - \langle \hat{H} \rangle^2}_{=0}} = 0$

$\int E \cdot 0 \, dx$

$0 \cdot 0$

$E \cdot E$

$\psi(x)$ and $V(x) \Rightarrow \hat{H}\psi(x) = E\psi(x)$

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right)\psi(x) = (\quad)\psi$$

$$[\hat{x}, \hat{p}^2] f(x) = 2\hbar^2 \frac{\partial}{\partial x} f(x)$$

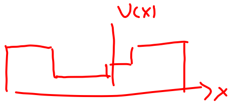
$[\hat{x}, \hat{p}] = i\hbar$ $\quad \hat{a}_+ = \quad \hat{p}$

$\hat{a}_- = \quad \hat{x}$

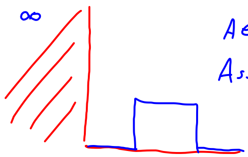
$\langle \hat{x}\hat{p} \rangle$ $\quad \langle \hat{x} \rangle, \langle \hat{p} \rangle, \langle \hat{x}^2 \rangle, \langle \hat{p}^2 \rangle$

V(x)

Piecewise Constant



∞



$A e^{ikx} + B e^{-ikx}$

$A \sin(kx) + B\cos(kx)$

continuous spectrum

$$\phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \psi^*(k,x,t)\, \Psi(x,0)\, dx$$

TISE $\Rightarrow$ $\psi_n(x)$ $\psi(k,x)$

apply b.c.s

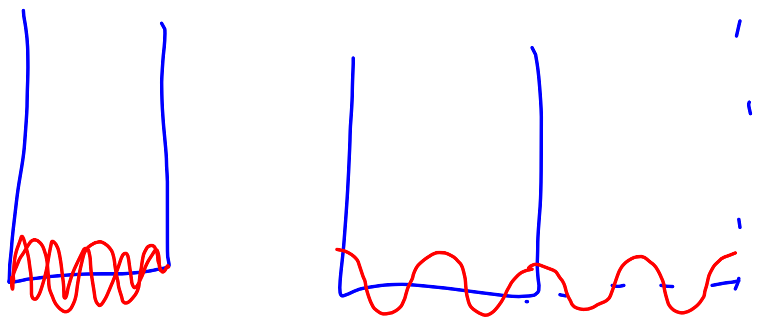
$\sin(kx)$
$\cos(kx)$ $\Rightarrow$ $k = \frac{2n\pi}{L}$

$$\psi_n(x) = A\cos\left(\frac{2n\pi}{L}x\right)$$

$$\underline{\Psi}(x,0) = A\cos\left(\frac{6\pi}{L}x\right)$$

$C_3 = 1$ $C_{n\neq 3} = 0$

$V(x) = \frac{1}{2}m\omega^2 x^2$



$$\hat{a}_+ = \frac{1}{\sqrt{2m\hbar\omega}}(\hat{x}m\omega - i\hat{p})$$

$[\hat{x},\hat{p}]$

$$\hat{a}_- = \frac{1}{\sqrt{2m\hbar\omega}}(\hat{x}m\omega + i\hat{p})$$

$S = \int L\,dt \qquad L = T - V = \int \left[\frac{1}{2}mv^2 - V(x)\right] dt$

$\delta S = \delta \int L\,dt = 0$

$x(t)$

$\dot{x}(t) = v(t)$

$f(x) = x^3$

$g(f(x))$

e.o.m.

Lagrange e.o.m.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial v}\right) - \frac{\partial L}{\partial x} = 0$$

$$PI = \left.\int e^{\frac{i}{\hbar}\int L\,dt}\,[dx(t)]\right\} \longrightarrow \text{T.D.S.E.}$$

E+M wave $\quad \dfrac{\partial^2 E}{\partial x^2} - \dfrac{1}{c^2}\dfrac{\partial^2 E}{\partial t^2} = 0 \Rightarrow E(x,t) = E_0 e^{i(kx - \omega t)}$

$$\frac{\partial^2 E}{\partial x^2} = -k^2 E(x,t)$$

$$\left(-k^2 + \frac{\omega^2}{c^2}\right) E(x,t) = 0 \qquad \frac{\partial^2 E}{\partial t^2} = -\omega^2 E(x,t)$$

$$k^2 = \frac{\omega^2}{c^2} \Rightarrow c = \frac{\omega}{k} = \lambda f$$

Consider: $\varepsilon = \hbar\omega, \; p = \hbar k$

$$\left[-\frac{p^2}{\hbar^2} - \frac{1}{c^2}\left(-\frac{\varepsilon^2}{\hbar^2}\right)\right] E(x,t) = 0$$

$$-\frac{1}{\hbar^2}\Big[\underbrace{p^2 - \frac{\varepsilon^2}{c^2}}\Big] E(x,t) = 0$$

$$\varepsilon^2 = p^2 c^2$$

$$\Big\downarrow \text{massive}$$

$$\varepsilon^2 = p^2 c^2 + m^2 c^4$$

Apply deBroglie $\quad E_0 \to \phi_0$

$$-\frac{1}{\hbar^2}\left[p^2 - \frac{\varepsilon^2}{c^2} + m^2 c^2\right] \phi_0 e^{\frac{i}{\hbar}(px - \varepsilon t)} = 0$$

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2 c^2}{\hbar^2}\right) \phi_0 e^{\frac{i}{\hbar}(px - \varepsilon t)} = 0$$

$$\frac{\partial^2 \phi}{\partial x^2} - \frac{m^2 c^2}{\hbar^2}\phi = \frac{1}{c^2}\frac{\partial^2 \phi}{\partial t^2} \qquad [\text{Klein-Gordon eqn.}]$$

relativistic SE

NR limit: $\varepsilon = mc^2 \sqrt{1 + \frac{p^2}{m^2 c^2}} \approx mc^2\left(1 + \frac{1}{2}\frac{p^2}{m^2 c^2}\right)$

$$= mc^2 + \frac{1}{2}\frac{p^2}{m}$$

$$= mc^2 + T$$

$$\phi(x,t) = \phi_0 e^{\frac{i}{\hbar}(px - Et)} = \phi_0 e^{\frac{i}{\hbar}(px - mc^2 t - Tt)}$$

$$= \underbrace{e^{-\frac{i}{\hbar}mc^2 t}}\, \phi_0 e^{\frac{i}{\hbar}(px - Tt)}$$

oscillates for relativistic system

Introduce: $\phi = e^{-\frac{i}{\hbar}mc^2 t}\, \psi(x,t)$

$$\frac{\partial^2 \phi}{\partial t^2} = -\frac{1}{\hbar^2} m^2 c^4 e^{-\frac{i}{\hbar}mc^2 t}\psi - \frac{2i}{\hbar} mc^2 e^{-\frac{i}{\hbar}mc^2 t}\frac{\partial \psi}{\partial t} + \cancel{e^{-\frac{i}{\hbar}mc^2 t}\frac{\partial^2 \psi}{\partial t^2}}$$

$$\left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{m^2 c^2}{\hbar^2}\right)\phi = 0$$

$$e^{-\frac{i}{\hbar}mc^2 t}\underbrace{\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{2im}{\hbar}\frac{\partial \psi}{\partial t}\right) = 0}_{\text{TDSE}}$$