Spin-1 $(s=1)$

$$\hat{S}^2 \chi_s^m = \hbar^2 s(s+1)\chi_s^m = 2\hbar^2 \chi_1^m$$

$$\hat{S}_z \chi_s^m = \hbar m \chi_s^m$$

$$\boxed{m = -1, 0, 1}$$

3 possible states: eigenspinors of S_z

$$\chi_1^{+1} = \begin{pmatrix}1\\0\\0\end{pmatrix} \qquad \chi_1^{0} = \begin{pmatrix}0\\1\\0\end{pmatrix} \qquad \chi_1^{-1} = \begin{pmatrix}0\\0\\1\end{pmatrix}$$

$$\hat{S}^2 = \begin{pmatrix}a & b & c\\ d & e & f\\ g & h & k\end{pmatrix}$$

$$\hat{S}^2 \chi_1^{+1} = 2\hbar^2 \chi_1^{+1} \qquad \hat{S}^2 \chi_1^{0} = 2\hbar^2 \chi_1^{0} \qquad \hat{S}^2 \chi_1^{-1} = 2\hbar^2 \chi_1^{-1}$$

$$\begin{pmatrix}a & b & c\\ d & e & f\\ g & h & k\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix} = 2\hbar^2 \begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\begin{pmatrix}a\\d\\g\end{pmatrix} = \begin{pmatrix}2\hbar^2\\0\\0\end{pmatrix} \qquad \begin{pmatrix}b\\e\\h\end{pmatrix} = \begin{pmatrix}0\\2\hbar^2\\0\end{pmatrix} \qquad \begin{pmatrix}c\\f\\k\end{pmatrix} = \begin{pmatrix}0\\0\\2\hbar^2\end{pmatrix}$$

$$\hat{S}^2 = 2\hbar^2 \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

$$\hat{S}_z = \begin{pmatrix}a & b & c\\ d & e & f\\ g & h & k\end{pmatrix}$$

$$\hat{S}_z \chi_1^{+1} = \hbar \chi_1^{+1} \qquad \hat{S}_z \chi_1^{0} = 0 \cdot \chi_1^{0} \qquad \hat{S}_z \chi_1^{-1} = -\hbar \chi_1^{-1}$$

$$\begin{pmatrix}a & b & c\\ d & e & f\\ g & h & k\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}\hbar\\0\\0\end{pmatrix}$$

$$\begin{pmatrix}a\\d\\g\end{pmatrix} = \begin{pmatrix}\hbar\\0\\0\end{pmatrix}$$

$$\hat{S}_z = \hbar \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}$$

$$\hat{S}_\pm = \hat{S}_x \pm i \hat{S}_y \Rightarrow$$

$$\hat{S}_+ \chi_1^{+1} = 0 \qquad \hat{S}_- \chi_1^{+1} = \hbar\sqrt{2}\,\chi_1^{0}$$

$$\hat{S}_+ \chi_1^{0} = \hbar\sqrt{2}\,\chi_1^{+1} \qquad \hat{S}_- \chi_1^{0} = \hbar\sqrt{2}\,\chi_1^{-1}$$

$$\hat{S}_+ \chi_1^{-1} = \hbar\sqrt{2}\,\chi_1^{0} \qquad \hat{S}_- \chi_1^{-1} = 0$$

$$A_l^m = \hbar\sqrt{l(l+1) - m(m\pm 1)}$$

$$A_s^m$$

$$\hat{S}_x = \frac{1}{2}(\hat{S}_+ + \hat{S}_-) = \sqrt{2}\hbar \begin{pmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\end{pmatrix}$$

$$\hat{S}_y = \frac{1}{2i}(\hat{S}_+ - \hat{S}_-) = \sqrt{2}\hbar \begin{pmatrix}0 & 0 & 0\\ 1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix}$$

$$\hat{\vec{S}} = \hat{S}_x \hat{i} + \hat{S}_y \hat{j} + \hat{S}_z \hat{k}$$

Total Wavefunction

$$\hat{S}^2 \psi = \psi_{nlm_l}(\hat{S}^2 \chi_s^{m_s})$$

$$\psi = \psi_{nlm_l}\chi_s^{m_s}$$

$$\psi_{example} = R_{21}(r)\, Y_1^0(\theta,\phi)\begin{pmatrix}1\\0\end{pmatrix}$$

$$\hat{H}\psi = (\hat{H}\psi_{nlm_l})\chi_s^{m_s}$$