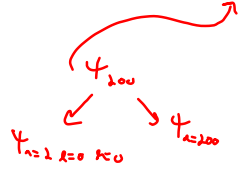


Review: $\hat{H}_{hyd} \Psi = E \Psi$ $\hat{H}_{hyd} = \frac{\hat{p}^2}{2m} - \frac{e^2}{4\pi\epsilon_0} \frac{1}{r}$

$\Psi(r, \theta, \phi, t) = \sum_n c_n e^{-i\frac{E_n}{\hbar}t} R_{nl}(r) Y_l^m(\theta, \phi)$ TISE
 $\Psi_{nem}(r, \theta, \phi)$ solves $\hat{H}_{hyd} \Psi = E_n \Psi_{nem}$
 generate an orthonormal basis in Hilbert space of bound states
 $\int \Psi_{nem}^* \Psi_{n'l'm'} dV = \delta_{nn'} \delta_{ll'} \delta_{mm'}$



To solve we first separated $R(r)$ from Θ, Φ using $l(l+1)$.

Y_l^m - known functions (spherical harmonics) - you give me a value of l, m I give $Y_l^m(\theta, \phi)$.

$R_{nl}(r)$ - learned how to build

$$R_{nl}(r) = \frac{u(r)}{r} = \frac{u(\rho)}{\rho k} = \frac{k}{\rho} (\rho^{2l+1} e^{-\rho} v(\rho)) = k \rho^l e^{-\rho} \sum_{j=0}^{\infty} c_{j+1} \rho^{j+1}$$

$$= k(kr)^l e^{-kr} \sum_{j=0}^{\infty} c_j \rho^j \quad (\text{give me an } l, \text{ finding } E \text{ and } c_j \text{'s is solving problem})$$

$k = \frac{\sqrt{2mE}}{\hbar}$

$$c_{j+1} = \frac{l(j+l+1-n)}{(j+1)(j+l+2)} c_j$$

$$E_n = - \left[\frac{n}{2a_0} \left(\frac{e^2}{4\pi\epsilon_0} \right) \right]^2 \frac{1}{2m} \quad (\text{given } k, \text{ find } E, \text{ and helps determine when } \sum \text{ terminates})$$

Choose n, l and start with an unknown c_0 :

$$c_1 = c_{0+1} = \frac{l(0+l+1-n)}{(0+1)(0+l+2)} c_0$$

$$c_2 = c_{1+1} = \frac{l(1+l+1-n)}{(1+1)(1+l+2)} c_1 = [\quad] [\quad] c_0$$

$$\vdots$$

$$c_{j_{max}+1} = \frac{l(j_{max}+l+1-n)}{\quad} c_{j_{max}} \quad \neq 0$$

$$0 \quad \parallel \quad j_{max}+l+1-n = 0$$

Degeneracy in l comes from fixing n (E_n) and finding all pairs of j_{max} and l which satisfy $j_{max} + l + 1 - n = 0$

Each l gives a different j_{max} and hence a different v_{ϕ} .

Question: Consider $l=1, n=3$ and $l=0, n=2$

Do these have the same ψ ?

$$v(\rho) = C_0 + C_1 \rho$$

$$C_1^{31} = -\frac{2}{a_1} \quad E_3 \Rightarrow K_3$$

$$C_1^{20} = -\frac{2}{2} \quad E_2 \Rightarrow K_2$$

also have ρ^2 in R_{nl}

furthermore ψ_L^n differ.

Just give the answer dermit!

$$v(\rho) = L_{n-l-1}^{2l+1}(2\rho)$$

$$L_{q-p}^p(x) \equiv (-1)^p \left(\frac{d}{dx}\right)^p L_q(x) \quad \text{associated Laguerre polynomials}$$

$$L_q(x) \equiv e^x \left(\frac{d}{dx}\right)^q (e^{-x} x^q) \quad q\text{-th order Laguerre polynomial}$$

Finally:

$$\psi_{nlm} = \sqrt{\frac{2}{\pi} \frac{(n-l)!}{(n+l)!}} e^{-r/nc} \left(\frac{dr}{nc}\right)^l L_{n-l-1}^{2l+1}\left(\frac{dr}{nc}\right) Y_l^m(\theta, \phi)$$

$$E_n = -\frac{k^2}{2nc^2} \frac{1}{n^2}$$

$$a \equiv \frac{4\pi\epsilon_0 \hbar^2}{me^2} = \text{Bohr radius}$$