

HW Quiz 2

Answer one of the following questions. If you answer both questions, you will be scored on each and you will receive the lower of the two.

- 1) Consider the time-dependent Schrodinger equation: $i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi$

Is it possible to consider only real solutions to this equation **without** losing generality? Explain your answer. **[No]** Conjugating the equation $-i\hbar \frac{\partial \Psi^*}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi^*}{\partial x^2} + V\Psi^*$ tells us that Ψ^* satisfies a different equation. So the real linear combinations $\Psi + \Psi^*$ and $i(\Psi - \Psi^*)$ do **not** satisfy the original equation.

2) The total initial wavefunction for a particle in an infinite well (from $x = 0$ to a) is

$$\Psi(x,0) = \frac{1}{\sqrt{3}}\Psi_2 + \frac{2}{\sqrt{3}}\Psi_5$$

Note: Ψ 's were **not** properly normalized!!

An orthonormal set of solutions to the T.I.S.E. for this system and their energies is given by:

$$\Psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right) \quad E_n = \frac{\hbar^2 \pi^2 n^2}{2ma^2}$$

- a) Evaluate $\int_0^a \Psi_2^*(x)\Psi(x,0)dx = \int_0^a \frac{1}{\sqrt{3}}\Psi_2^* \left(\frac{1}{\sqrt{3}}\Psi_2 + \frac{2}{\sqrt{3}}\Psi_5 \right) dx = \frac{1}{\sqrt{3}} \int_0^a \Psi_2^* \Psi_2 dx + \frac{2}{\sqrt{3}} \int_0^a \Psi_2^* \Psi_5 dx = \frac{1}{\sqrt{3}} \int_0^a \Psi_2^* \Psi_2 dx = \frac{1}{\sqrt{3}}$

- b) Write down an expression for $\Psi(x,t)$. **Hint: Recall that $\phi_n(t) = e^{-i\frac{E_n}{\hbar}t}$**
- c) Calculate $\langle \hat{H} \rangle$ for this total wavefunction.

$$\langle \hat{H} \rangle = \frac{1}{3}E_2 + \frac{4}{3}E_5$$

$$\Psi(x,t) = \frac{1}{\sqrt{3}}\Psi_2 e^{-i\frac{E_2}{\hbar}t} + \frac{2}{\sqrt{3}}\Psi_5 e^{-i\frac{E_5}{\hbar}t}$$