HW Quiz 3

Work one of the following problems **(You must show your work)**:

1. Consider the simple harmonic oscillator.

   a) Find <x> for the $(n+2)th$ state. $\sqrt{\frac{\hbar}{2m\omega}}\int_{-\infty}^{\infty}\psi^*_{n+2}(\hat{a}_+ + \hat{a}_-)\psi_{n+2}\,dx = \sqrt{\frac{\hbar}{2m\omega}}\left\{\sqrt{n+3}\int_{-\infty}^{\infty}\cancelto{0}{\psi^*_{n+2}\psi_{n+3}}\,dx + \sqrt{n+2}\int_{-\infty}^{\infty}\cancelto{0}{\psi^*_{n+2}\psi_{n+1}}\,dx\right\}$

   $\boxed{\langle x\rangle = 0}$

   b) Find <p²> for the $(n+2)th$ state.

$$\langle p^2\rangle = -\frac{\hbar m\omega}{2}\int_{-\infty}^{\infty}\psi^*_{n+2}(\cancelto{0}{\hat{a}_+\hat{a}_+} + \cancelto{0}{\hat{a}_-\hat{a}_-} - \hat{a}_+\hat{a}_- - \hat{a}_-\hat{a}_+)\psi_{n+2}\,dx$$

$$= \frac{\hbar m\omega}{2}\left\{(n+2)\int_{-\infty}^{\infty}\cancelto{1}{\psi^*_{n+2}\psi_{n+2}}\,dx + (n+3)\int_{-\infty}^{\infty}\cancelto{1}{\psi^*_{n+2}\psi_{n+2}}\,dx\right\} = \boxed{\frac{\hbar m\omega}{2}(2n+5) = \langle p^2\rangle}$$

2. A free particle has the initial wavefunction

$$\Psi(x,0) = Ae^{-a|x|}$$

   where A and a are positive real constants.

   a) Find A. $\int_{-\infty}^{\infty}\Psi^*\Psi\,dx = 1 = A^2 2\int_0^{\infty}e^{-2ax}\,dx = \frac{-2A^2}{2a}e^{-2ax}\Big|_0^{\infty} = \frac{A^2}{a} \Rightarrow A = \sqrt{a}$

   b) Find $\varphi(k)$. $\phi(k) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}e^{ikx}\sqrt{a}\,e^{-a|x|}\,dx = \sqrt{\frac{a}{2\pi}}\int_{-\infty}^{\infty}\Big[\underset{E}{\cos(kx)}\,\underset{E}{e^{-a|x|}} + \cancelto{0}{i\,\underset{O}{\sin(kx)}\,\underset{E}{e^{-a|x|}}}\Big]dx$

$$= 2\sqrt{\frac{a}{2\pi}}\int_0^{\infty}\cos(kx)e^{-ax}\,dx = 2\sqrt{\frac{a}{2\pi}}\int_0^{\infty}(e^{ikx} - e^{-ikx})e^{-ax}\,dx = 2\sqrt{\frac{a}{2\pi}}\int_0^{\infty}\left[e^{(ik-a)x} - e^{-(ik+a)x}\right]dx$$

$$= 2\sqrt{\frac{a}{2\pi}}\left[\frac{-1}{ik-a} - \frac{+1}{ik+a}\right]$$