HW Quiz 6

Work one of the following problems (show your work):

1. Calculate $\frac{d\langle \hat{p}\hat{H} \rangle}{dt}$ for a free particle.

$\frac{d\langle \hat{p}\hat{H} \rangle}{dt} = \frac{i}{\hbar}\langle [\hat{H}, \hat{p}\hat{H}] \rangle + \langle \frac{\partial \hat{p}\hat{H}}{\partial t} \rangle$

$\hat{H} = \frac{\hat{p}^2}{2m}$ for free particle so $[\frac{\hat{p}^2}{2m}, \frac{\hat{p}^2}{2m}] = 0$, $\frac{\partial \hat{p}\hat{H}}{\partial t} = 0$

Then $\boxed{\frac{d\langle \hat{p}\hat{H} \rangle}{dt} = 0}$

2. Starting from the expression: $\frac{d}{dt}\langle \hat{x}\hat{p} \rangle = 2\langle \hat{T} \rangle - \left\langle \hat{x}\frac{d\hat{V}}{d\hat{x}} \right\rangle$

find the proportionality constant *k* in the "Virial-like" relation $\langle \hat{T} \rangle = k\langle \hat{V} \rangle$ for a stationary state of the quartic oscillator governed by $V(x) = m\alpha^2 x^4$.

any operator

For stationary states: $\frac{d\langle \hat{Q} \rangle}{dt} = 0 \Rightarrow \frac{d\langle \hat{x}\hat{p} \rangle}{dt} = 0 \Rightarrow 2\langle \hat{T} \rangle = \langle \hat{x}\frac{d\hat{V}}{d\hat{x}} \rangle$

For quartic oscillator: $\frac{d\hat{V}}{d\hat{x}} = 4m\alpha^2\hat{x}^3 \Rightarrow 2\langle \hat{T} \rangle = \langle 4m\alpha^2\hat{x}^4 \rangle = 4\langle \hat{V} \rangle \Rightarrow \boxed{\langle \hat{T} \rangle = 2\langle \hat{V} \rangle}$