# HW Quiz 7

Work one of the following problems (show your work):

1. Calculate the degeneracy of the 10th energy level of a particle in a 3D infinite cubical well.

$E_n = E_{n_x n_y n_z} \propto (n_x^2 + n_y^2 + n_z^2)$ ← overall level

$E_1 = E_{111} \propto 3$ $E_6 = E_{223} \propto 14$
$E_2 = E_{112} \propto 6$ $E_7 = E_{223} \propto 17$
$E_3 = E_{113} \propto 9$ $E_8 = E_{134} \propto 18$
$E_4 = E_{117} \propto 11$ $E_9 = E_{133} \propto 19$
$E_5 = E_{222} \propto 12$ $E_{10} = E_{134} \propto 21$ w/ permutations: $(1\,2\,4)(1\,4\,2)(2\,1\,4)(2\,4\,1)(4\,1\,2)(4\,2\,1)$ so $d_{10} = 6$

2. Given:

$$P_l^m(x) \equiv (1-x^2)^{|m|/2}\left(\frac{d}{dx}\right)^{|m|} P_l(x) \qquad P_l(x) \equiv \frac{1}{2^l l!}\left(\frac{d}{dx}\right)^l (x^2-1)^l$$

construct $P_5^{-5}(\cos\theta)$. Your answer should **not** contain derivatives!

$$P_5^{-5}(x) = (1-x^2)^{5/2}\left(\frac{d}{dx}\right)^5 \frac{1}{2^5 5!}\left(\frac{d}{dx}\right)^5 (x^2-1)^5 = (1-x^2)^{5/2}\frac{1}{2^5 5!}\underbrace{\left(\frac{d}{dx}\right)^{10}}_{10!}(x^{10} + \cdots)$$

rest of terms will have $x^n$ w/ $n < 10$ and will not survive $\left(\frac{d}{dx}\right)^{10}$.

$$= (1-x^2)^{5/2}\frac{1\cdot 10!}{2^5 5!} \Rightarrow P_5^{-5}(\cos\theta) = (1-\cos^2\theta)^{5/2}\frac{10!}{2^5 5!}$$