

HW Quiz 8

Work one of the following problems (show your work):

1. For a state ψ satisfying $\hat{L}^2\psi = \hbar^2 a \psi$ and $\hat{L}_z\psi = \hbar b \psi$ and a second state ψ' for which $\langle \psi' | \psi \rangle = 7$ evaluate $\langle L_- \psi' | L_- \psi \rangle$.

$$\begin{aligned}
 L^2 &= L_+ L_- + L_z^2 + \hbar L_z & \langle L_- \psi' | L_- \psi \rangle &= \langle \psi' | L_+ L_- \psi \rangle \\
 L_+ L_- &= L^2 - L_z^2 + \hbar L_z & &= \langle \psi' | (L^2 - L_z^2 + \hbar L_z) \psi \rangle \\
 & & &= (\hbar^2 a - \hbar^2 b^2 + \hbar^2 b) \langle \psi' | \psi \rangle \\
 & & &= \boxed{7(\hbar^2 a - \hbar^2 b^2 + \hbar^2 b)}
 \end{aligned}$$

2. Show that $[L_z, r] = 0$.

$$\hat{L}_z = \hat{x} \hat{p}_y - \hat{y} \hat{p}_x$$

$$\begin{aligned}
 [x p_y - y p_x, (x^2 + y^2 + z^2)^{\frac{1}{2}}] &= [x(-i\hbar \frac{\partial}{\partial y}) - y(-i\hbar \frac{\partial}{\partial x}), (x^2 + y^2 + z^2)^{\frac{1}{2}}] f \\
 &= (x^2 + y^2 + z^2)^{\frac{1}{2}} [x(-i\hbar \frac{\partial}{\partial y}) - y(-i\hbar \frac{\partial}{\partial x})] f \\
 &= -i\hbar f (x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x}) (x^2 + y^2 + z^2)^{\frac{1}{2}} = -i\hbar f (x \frac{y}{\sqrt{x^2 + y^2 + z^2}} - y \frac{x}{\sqrt{x^2 + y^2 + z^2}}) = 0
 \end{aligned}$$

product rule (arrow from $(x^2 + y^2 + z^2)^{\frac{1}{2}}$ to the operator)

when [] acts on f it will create a term to cancel off (arrow from the operator to the f term)