HW Quiz 8

Work one of the following problems (show your work):

1. For a state ψ satisfying $\hat{L}^2\psi = \hbar^2 a\psi$ and $\hat{L}_z\psi = \hbar b\psi$ and a second state ψ' for which $\langle \psi' | \psi \rangle = 7$ evaluate $\langle L_-\psi' | L_-\psi \rangle$.

$$L^2 = L_\pm L_\mp + L_z^2 \mp \hbar L_z \qquad \langle L_-\psi' | L_-\psi \rangle = \langle \psi' | L_+ L_- \psi \rangle$$

$$L_+ L_- = L^2 - L_z^2 + \hbar L_z$$

$$= \langle \psi' | (L^2 - L_z^2 + \hbar L_z)\psi \rangle$$

$$= (\hbar^2 a - \hbar^2 b^2 + \hbar^2 b)\langle \psi' | \psi \rangle$$

$$= \boxed{7(\hbar^2 a - \hbar^2 b^2 + \hbar^2 b)}$$

2. Show that $[L_z, r] = 0$.

$$\hat{L}_z = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x$$

product rule

when [ ] acts on f it will create a term to cancel off

$$[xp_y - yp_x, (x^2+y^2+z^2)^{\frac{1}{2}}] = \left[x\left(-i\hbar\frac{\partial}{\partial y}\right) - y\left(-i\hbar\frac{\partial}{\partial x}\right)\right](x^2+y^2+z^2)^{\frac{1}{2}} f$$

$$- (x^2+y^2+z^2)^{\frac{1}{2}}\left[x\left(-i\hbar\frac{\partial}{\partial y}\right) - y\left(-i\hbar\frac{\partial}{\partial x}\right)\right] f$$

$$= -i\hbar f\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)(x^2+y^2+z^2)^{\frac{1}{2}} = -i\hbar f\left(x\frac{y}{\sqrt{x^2+y^2+z^2}} - y\frac{x}{\sqrt{x^2+y^2+z^2}}\right) = 0$$