HW Quiz 9

Work one of the following problems (show your work):

$S^2 = S_1^2 + S_2^2 + 2S_{1x}S_{2x} + 2S_{1y}S_{2y} + 2S_{1z}S_{2z}$

1. For a system of two spin-1/2 particles, show that the spin singlet state $|00\rangle$ is an eigenstate of $\hat{S}^2$ with eigenvalue 0.

$S^2\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) = \frac{1}{\sqrt{2}}\Big[\underbrace{(S_1^2\uparrow)}_{\frac{3}{4}\hbar^2\uparrow}\downarrow - \underbrace{(S_1^2\downarrow)}_{\frac{3}{4}\hbar^2\downarrow}\uparrow + \uparrow\underbrace{(S_2^2\downarrow)}_{\frac{3}{4}\hbar^2\downarrow} - \downarrow\underbrace{(S_2^2\uparrow)}_{\frac{3}{4}\hbar^2\uparrow} + 2\Big\{\underbrace{(S_{1x}\uparrow)}_{\frac{\hbar}{2}\downarrow}\underbrace{(S_{2x}\downarrow)}_{\frac{\hbar}{2}\uparrow} - \underbrace{(S_{1x}\downarrow)}_{\frac{\hbar}{2}\uparrow}\underbrace{(S_{2x}\uparrow)}_{\frac{\hbar}{2}\downarrow} + \underbrace{(S_{1y}\uparrow)}_{\frac{i\hbar}{2}\downarrow}\underbrace{(S_{2y}\downarrow)}_{-\frac{i\hbar}{2}\uparrow} - \underbrace{(S_{1y}\downarrow)}_{-\frac{i\hbar}{2}\uparrow}\underbrace{(S_{2y}\uparrow)}_{\frac{i\hbar}{2}\downarrow} + \underbrace{(S_{1z}\uparrow)}_{\frac{\hbar}{2}\uparrow}\underbrace{(S_{2z}\downarrow)}_{-\frac{\hbar}{2}\downarrow} - \underbrace{(S_{1z}\downarrow)}_{-\frac{\hbar}{2}\downarrow}\underbrace{(S_{2z}\uparrow)}_{\frac{\hbar}{2}\uparrow}\Big\}\Big]$

$= \frac{1}{\sqrt{2}}\Big[\left(\frac{3}{4}\hbar^2 + \frac{3}{4}\hbar^2 - \frac{\hbar^2}{2} - \frac{\hbar^2}{2} - \frac{\hbar^2}{2}\right)\uparrow\downarrow - \left(\frac{3}{4}\hbar^2 + \frac{3}{4}\hbar^2 - \frac{\hbar^2}{2} - \frac{\hbar^2}{2} - \frac{\hbar^2}{2}\right)\downarrow\uparrow\Big] = \boxed{0\,\frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)}$

2. For two identical fermions in a SHO potential, write an integral expression for $\left\langle (x_1 - x_2)^2 \right\rangle - \left\langle x^2 \right\rangle_{n=0} - \left\langle x^2 \right\rangle_{n=1} + 2\langle x \rangle_{n=0}\langle x \rangle_{n=1}$ if the *n=0* and *n=1* wavefunctions are given by:

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2} \qquad \psi_1(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\sqrt{\frac{2m\omega}{\hbar}}\, x e^{-\frac{m\omega}{2\hbar}x^2}$$

NOTE: Your answer should involve only one integral and you do not have to evaluate it! From book: $\langle (x_1 - x_2)^2 \rangle_\pm = \langle x^2 \rangle_a + \langle x^2 \rangle_b - 2\langle x \rangle_a \langle x \rangle_b \mp 2|\langle x \rangle_{ab}|^2$

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Then: $\langle (x_1 - x_2)^2 \rangle - \langle x^2 \rangle_a - \langle x^2 \rangle_b + 2\langle x \rangle_a \langle x \rangle_b = 2|\langle x \rangle_{ab}|^2$ for fermions $= 2\left|\int x\psi_a^*\psi_b\,dx\right|^2 = 2\left|\int x^2\left(\frac{m\omega}{\pi\hbar}\right)^{\frac{1}{2}}\sqrt{\frac{2m\omega}{\hbar}}\, e^{-\frac{m\omega}{\hbar}x^2}\right|^2$