Problem 5.22

(a)

$$\psi(x_A, x_B, x_C) = \frac{1}{\sqrt{6}}\left(\sqrt{\frac{2}{a}}\right)^3 \left[\sin\left(\frac{5\pi x_A}{a}\right)\sin\left(\frac{7\pi x_B}{a}\right)\sin\left(\frac{17\pi x_C}{a}\right) - \sin\left(\frac{5\pi x_A}{a}\right)\sin\left(\frac{17\pi x_B}{a}\right)\sin\left(\frac{7\pi x_C}{a}\right)\right.$$
$$+\sin\left(\frac{7\pi x_A}{a}\right)\sin\left(\frac{17\pi x_B}{a}\right)\sin\left(\frac{5\pi x_C}{a}\right) - \sin\left(\frac{7\pi x_A}{a}\right)\sin\left(\frac{5\pi x_B}{a}\right)\sin\left(\frac{17\pi x_C}{a}\right)$$
$$\left.+\sin\left(\frac{17\pi x_A}{a}\right)\sin\left(\frac{5\pi x_B}{a}\right)\sin\left(\frac{7\pi x_C}{a}\right) - \sin\left(\frac{17\pi x_A}{a}\right)\sin\left(\frac{7\pi x_B}{a}\right)\sin\left(\frac{5\pi x_C}{a}\right)\right].$$

(b) (i)

$$\psi = \left(\sqrt{\frac{2}{a}}\right)^3 \left[\sin\left(\frac{11\pi x_A}{a}\right)\sin\left(\frac{11\pi x_B}{a}\right)\sin\left(\frac{11\pi x_C}{a}\right)\right].$$

(ii)

$$\psi = \frac{1}{\sqrt{3}}\left(\sqrt{\frac{2}{a}}\right)^3 \left[\sin\left(\frac{\pi x_A}{a}\right)\sin\left(\frac{\pi x_B}{a}\right)\sin\left(\frac{19\pi x_C}{a}\right) + \sin\left(\frac{\pi x_A}{a}\right)\sin\left(\frac{19\pi x_B}{a}\right)\sin\left(\frac{\pi x_C}{a}\right) + \sin\left(\frac{19\pi x_A}{a}\right)\sin\left(\frac{\pi x_B}{a}\right)\sin\left(\frac{\pi x_C}{a}\right)\right].$$

(iii)

$$\psi = \frac{1}{\sqrt{6}}\left(\sqrt{\frac{2}{a}}\right)^3 \left[\sin\left(\frac{5\pi x_A}{a}\right)\sin\left(\frac{7\pi x_B}{a}\right)\sin\left(\frac{17\pi x_C}{a}\right) + \sin\left(\frac{5\pi x_A}{a}\right)\sin\left(\frac{17\pi x_B}{a}\right)\sin\left(\frac{7\pi x_C}{a}\right) + \sin\left(\frac{7\pi x_A}{a}\right)\sin\left(\frac{17\pi x_B}{a}\right)\sin\left(\frac{5\pi x_C}{a}\right) + \sin\left(\frac{7\pi x_A}{a}\right)\sin\left(\frac{5\pi x_B}{a}\right)\sin\left(\frac{17\pi x_C}{a}\right) + \sin\left(\frac{17\pi x_A}{a}\right)\sin\left(\frac{5\pi x_B}{a}\right)\sin\left(\frac{7\pi x_C}{a}\right) + \sin\left(\frac{17\pi x_A}{a}\right)\sin\left(\frac{7\pi x_B}{a}\right)\sin\left(\frac{5\pi x_C}{a}\right)\right].$$

Problem 5.23

(a) $E_{n_1n_2n_3} = (n_1 + n_2 + n_3 + \frac{3}{2})\hbar\omega = \frac{9}{2}\hbar\omega \Rightarrow n_1 + n_2 + n_3 = 3.$ $(n_1, n_2, n_3 = 0, 1, 2, 3\dots).$

State n_1	n_2	n_3	Configuration $(N_0, N_1, N_2\dots)$	# of States
0	0	3		
0	3	0	(2,0,0,1,0,0 ...)	3
3	0	0		
0	1	2		
0	2	1		
1	0	2	(1,1,1,0,0,0 ...)	6
1	2	0		
2	0	1		
2	1	0		
1	1	1	(0,3,0,0,0 ...)	1

Possible single-particle energies:

$E_0 = \hbar\omega/2 : P_0 = 12/30 = 4/10.$
$E_1 = 3\hbar\omega/2 : P_1 = 9/30 = 3/10.$
$E_2 = 5\hbar\omega/2 : P_2 = 6/30 = 2/10.$
$E_3 = 7\hbar\omega/2 : P_3 = 3/30 = 1/10.$

Most probable configuration: (1,1,1,0,0,0 ...).

Most probable single-particle energy: $E_0 = \frac{1}{2}\hbar\omega.$

(b) For identical fermions the *only* configuration is (1,1,1,0,0,0 ...) (one state), so this is also the most probable configuration. The possible one-particle energies are

$E_0\ (P_0 = 1/3),\quad E_1\ (P_1 = 1/3),\quad E_2\ (P_2 = 1/3),$

and they are all equally likely, so it's a 3-way tie for the most probable energy.

(c) For identical bosons all three configurations are possible, and there is one state for each. Possible one-particle energies: $E_0(P_0 = 1/3), E_1(P_1 = 4/9), E_2(P_2 = 1/9), E_3(P_3 = 1/9).$ Most probable energy: $E_1.$

Problem 5.24

Here $N = 3$, and $d_n = 1$ for all states, so:

$$\begin{cases} \text{Eq. 5.74} \Rightarrow Q = 6\prod_{n=1}^{\infty}\dfrac{1}{N_n!} & \text{(distinguishable)}, \\ \text{Eq. 5.75} \Rightarrow Q = \prod_{n=1}^{\infty}\dfrac{1}{N_n!(1-N_n)!} & \text{(fermions)}, \\ \text{Eq. 5.77} \Rightarrow Q = 1 & \text{(bosons)}. \end{cases}$$

(In the products, *most* factors are $1/0!$ or $1/1!$, both of which are 1, so I won't write them.)

Configuration 1 ($N_{11} = 3$, others 0):
$$\begin{cases} Q = 6 \times \dfrac{1}{3!} = \boxed{1} & \text{(distinguishable)}, \\ Q = \dfrac{1}{3!} \times \dfrac{1}{(-2)!} = \boxed{0} & \text{(fermions)}, \\ Q = \boxed{1} & \text{(bosons)}. \end{cases}$$

Configuration 2 ($N_5 = 1, N_{13} = 2$):
$$\begin{cases} Q = 6 \times \dfrac{1}{1!} \times \dfrac{1}{2!} = \boxed{3} & \text{(distinguishable)}, \\ Q = \dfrac{1}{1!0!} \times \dfrac{1}{2!(-1)!} = \boxed{0} & \text{(fermions)}, \\ Q = \boxed{1} & \text{(bosons)}. \end{cases}$$

Configuration 3 ($N_1 = 2, N_{19} = 1$):
$$\begin{cases} Q = 6 \times \dfrac{1}{2!} \times \dfrac{1}{1!} = \boxed{3} & \text{(distinguishable)}, \\ Q = \dfrac{1}{2!(-1)!} \times \dfrac{1}{1!0!} = \boxed{0} & \text{(fermions)}, \\ Q = \boxed{1} & \text{(bosons)}. \end{cases}$$

Configuration 4 ($N_5 = N_7 = N_{17} = 1$):
$$\begin{cases} Q = 6 \times \dfrac{1}{1!} \times \dfrac{1}{1!} \times \dfrac{1}{1!} = \boxed{6} & \text{(distinguishable)}, \\ Q = \dfrac{1}{1!0!} \times \dfrac{1}{1!0!} \times \dfrac{1}{1!0!} = \boxed{1} & \text{(fermions)}, \\ Q = \boxed{1} & \text{(bosons)}. \end{cases}$$

All of these agree with what we got "by hand" at the top of page 231.