

**Problem 2.34**

(a)

$$\psi = \begin{cases} Ae^{ikx} + Be^{-ikx} & (x < 0) \\ Fe^{-\kappa x} & (x > 0) \end{cases} \text{ where } k = \frac{\sqrt{2mE}}{\hbar}; \kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar}.$$

- (1) Continuity of  $\psi$ :  $A + B = F$ .  
 (2) Continuity of  $\psi'$ :  $ik(A - B) = -\kappa F$ .

$$\Rightarrow A + B = -\frac{ik}{\kappa}(A - B) \Rightarrow A \left(1 + \frac{ik}{\kappa}\right) = -B \left(1 - \frac{ik}{\kappa}\right).$$

$$R = \left|\frac{B}{A}\right|^2 = \frac{|(1 + ik/\kappa)|^2}{|(1 - ik/\kappa)|^2} = \frac{1 + (k/\kappa)^2}{1 + (k/\kappa)^2} = \boxed{1}.$$

Although the wave function penetrates into the barrier, it is eventually all reflected.

(b)

$$\psi = \begin{cases} Ae^{ikx} + Be^{-ikx} & (x < 0) \\ Fe^{ilx} & (x > 0) \end{cases} \text{ where } k = \frac{\sqrt{2mE}}{\hbar}; l = \frac{\sqrt{2m(E - V_0)}}{\hbar}.$$

- (1) Continuity of  $\psi$ :  $A + B = F$ .  
 (2) Continuity of  $\psi'$ :  $ik(A - B) = ilF$ .

$$\Rightarrow A + B = \frac{k}{l}(A - B); A \left(1 - \frac{k}{l}\right) = -B \left(1 + \frac{k}{l}\right).$$

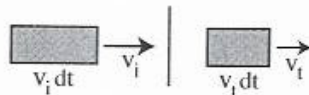
$$R = \left|\frac{B}{A}\right|^2 = \frac{(1 - k/l)^2}{(1 + k/l)^2} = \frac{(k - l)^2}{(k + l)^2} = \frac{(k - l)^4}{(k^2 - l^2)^2}.$$

$$\text{Now } k^2 - l^2 = \frac{2m}{\hbar^2}(E - E + V_0) = \left(\frac{2m}{\hbar^2}\right)V_0; k - l = \frac{\sqrt{2m}}{\hbar}|\sqrt{E} - \sqrt{E - V_0}|. \text{ so}$$

$$R = \frac{(\sqrt{E} - \sqrt{E - V_0})^4}{V_0^2}.$$

©2005 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This material is protected under all copyright laws as they currently exist. No portion of this material may be reproduced, in any form or by any means, without permission in writing from the publisher.

(c)



From the diagram,  $T = P_t/P_i = |F|^2 v_t / |A|^2 v_i$ , where  $P_i$  is the probability of finding the incident particle in the box corresponding to the time interval  $dt$ , and  $P_t$  is the probability of finding the transmitted particle in the associated box to the right of the barrier.

But  $\frac{v_t}{v_i} = \frac{\sqrt{E - V_0}}{\sqrt{E}}$  (from Eq. 2.98). So  $T = \sqrt{\frac{E - V_0}{E}} \left|\frac{F}{A}\right|^2$ . Alternatively, from Problem 2.19:

$$J_i = \frac{\hbar k}{m}|A|^2; J_t = \frac{\hbar l}{m}|F|^2; T = \frac{J_t}{J_i} = \left|\frac{F}{A}\right|^2 \frac{l}{k} = \left|\frac{F}{A}\right|^2 \sqrt{\frac{E - V_0}{E}}.$$

For  $E < V_0$ , of course,  $T = 0$ .

(d)

$$\text{For } E > V_0, F = A + B = A + A \left(\frac{k}{l} - 1\right) = A \frac{2k/l}{(k/l) + 1} = \frac{2k}{k + l} A.$$

$$T = \left|\frac{F}{A}\right|^2 \frac{l}{k} = \left(\frac{2k}{k + l}\right)^2 \frac{l}{k} = \frac{4kl}{(k + l)^2} = \frac{4kl(k - l)^2}{(k^2 - l^2)^2} = \frac{4\sqrt{E}\sqrt{E - V_0}(\sqrt{E} - \sqrt{E - V_0})^2}{V_0^2}.$$

$$T + R = \frac{4kl}{(k + l)^2} + \frac{(k - l)^2}{(k + l)^2} = \frac{4kl + k^2 - 2kl + l^2}{(k + l)^2} = \frac{k^2 + 2kl + l^2}{(k + l)^2} = \frac{(k + l)^2}{(k + l)^2} = 1. \checkmark$$