

Problem 2.23

(a)

$$(-2)^3 - 3(-2)^2 + 2(-2) - 1 = -8 - 12 - 4 - 1 = \boxed{-25}.$$

(b)

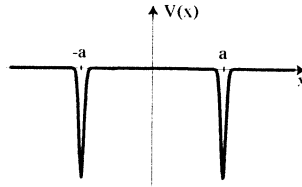
$$\cos(3\pi) + 2 = -1 + 2 = \boxed{1}.$$

(c)

$$\boxed{0} \text{ (} x = 2 \text{ is outside the domain of integration).}$$

Problem 2.27

(a)



(b) From Problem 2.1(c) the solutions are even or odd. Look first for *even solutions*:

$$\psi(x) = \begin{cases} Ae^{-\kappa x} & (x < -a), \\ B(e^{\kappa x} + e^{-\kappa x}) & (-a < x < a), \\ Ae^{\kappa x} & (x > a). \end{cases}$$

Continuity at a : $Ae^{-\kappa a} = B(e^{\kappa a} + e^{-\kappa a})$, or $A = B(e^{2\kappa a} + 1)$.

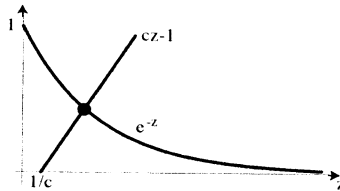
Discontinuous derivative at a , $\Delta \frac{d\psi}{dx} = -\frac{2m\alpha}{\hbar^2} \psi(a)$:

$$-\kappa Ae^{-\kappa a} - B(\kappa e^{\kappa a} - \kappa e^{-\kappa a}) = -\frac{2m\alpha}{\hbar^2} Ae^{-\kappa a} \Rightarrow A + B(e^{2\kappa a} - 1) = \frac{2m\alpha}{\hbar^2 \kappa} A; \text{ or}$$

$$B(e^{2\kappa a} - 1) = A \left(\frac{2m\alpha}{\hbar^2 \kappa} - 1 \right) = B(e^{2\kappa a} + 1) \left(\frac{2m\alpha}{\hbar^2 \kappa} - 1 \right) \Rightarrow e^{2\kappa a} - 1 = e^{2\kappa a} \left(\frac{2m\alpha}{\hbar^2 \kappa} - 1 \right) + \frac{2m\alpha}{\hbar^2 \kappa} - 1.$$

$$1 = \frac{2m\alpha}{\hbar^2 \kappa} - 1 + \frac{2m\alpha}{\hbar^2 \kappa} e^{-2\kappa a}; \quad \frac{\hbar^2 \kappa}{m\alpha} = 1 + e^{-2\kappa a}, \text{ or } \boxed{e^{-2\kappa a} = \frac{\hbar^2 \kappa}{m\alpha} - 1.}$$

This is a transcendental equation for κ (and hence for E). I'll solve it graphically: Let $z \equiv 2\kappa a$, $c \equiv \frac{\hbar^2}{2am\alpha}$, so $e^{-z} = cz - 1$. Plot both sides and look for intersections:



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From the graph, noting that c and z are both positive, we see that there is one (and only one) solution (for even ψ). If $\alpha = \frac{\hbar^2}{2ma}$, so $c = 1$, the calculator gives $z = 1.278$, so $\kappa^2 = -\frac{2mE}{\hbar^2} = \frac{z^2}{(2a)^2} \Rightarrow E = -\frac{(1.278)^2}{8} \left(\frac{\hbar^2}{ma^2}\right) = -0.204 \left(\frac{\hbar^2}{ma^2}\right)$.

Now look for *odd solutions*:

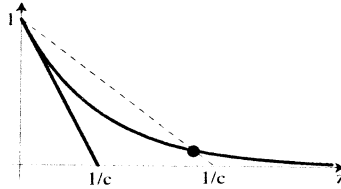
$$\psi(x) = \begin{cases} Ae^{-\kappa x} & (x < a), \\ B(e^{\kappa x} - e^{-\kappa x}) & (-a < x < a), \\ -Ae^{\kappa x} & (x < -a). \end{cases}$$

Continuity at a : $Ae^{-\kappa a} = B(e^{\kappa a} - e^{-\kappa a})$, or $A = B(e^{2\kappa a} - 1)$.

Discontinuity in ψ' : $-\kappa Ae^{-\kappa a} - B(\kappa e^{\kappa a} + \kappa e^{-\kappa a}) = -\frac{2m\alpha}{\hbar^2} Ae^{-\kappa a} \Rightarrow B(e^{2\kappa a} + 1) = A \left(\frac{2m\alpha}{\hbar^2 \kappa} - 1\right)$,

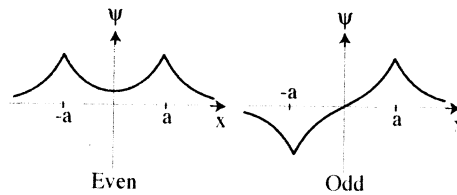
$$e^{2\kappa a} + 1 = (e^{2\kappa a} - 1) \left(\frac{2m\alpha}{\hbar^2 \kappa} - 1\right) = e^{2\kappa a} \left(\frac{2m\alpha}{\hbar^2 \kappa} - 1\right) - \frac{2m\alpha}{\hbar^2 \kappa} + 1,$$

$$1 = \frac{2m\alpha}{\hbar^2 \kappa} - 1 - \frac{2m\alpha}{\hbar^2 \kappa} e^{-2\kappa a}; \quad \frac{\hbar^2 \kappa}{m\alpha} = 1 - e^{-2\kappa a}, \quad \boxed{e^{-2\kappa a} = 1 - \frac{\hbar^2 \kappa}{m\alpha}}, \text{ or } e^{-z} = 1 - cz.$$



This time there may or may not be a solution. Both graphs have their y -intercepts at 1, but if c is too large (α too small), there may be no intersection (solid line), whereas if c is smaller (dashed line) there will be. (Note that $z = 0 \Rightarrow \kappa = 0$ is *not* a solution, since ψ is then non-normalizable.) The slope of e^{-z} (at $z = 0$) is -1 ; the slope of $(1 - cz)$ is $-c$. So there is an *odd solution* $\Leftrightarrow c < 1$, or $\alpha > \hbar^2/2ma$.

Conclusion: $One bound state if $\alpha \leq \hbar^2/2ma$; two if $\alpha > \hbar^2/2ma$.$



$$\alpha = \frac{\hbar^2}{ma} \Rightarrow c = \frac{1}{2} \cdot \begin{cases} \text{Even: } e^{-z} = \frac{1}{2}z - 1 \Rightarrow z = 2.21772, \\ \text{Odd: } e^{-z} = 1 - \frac{1}{2}z \Rightarrow z = 1.59362. \end{cases}$$

$$\boxed{E = -0.615(\hbar^2/ma^2); E = -0.317(\hbar^2/ma^2).}$$

$$\alpha = \frac{\hbar^2}{4ma} \Rightarrow c = 2. \text{ Only even: } e^{-z} = 2z - 1 \Rightarrow z = 0.738835; \quad \boxed{E = -0.0682(\hbar^2/ma^2).}$$