GPGN 404
Final Exam
December 13, 2005 **Name:** ____________________

Question:	1	2	3	4	Total
Points:	10	14	12	14	50
Score:					

Question 1 .... (10 points)

When applied to an input signal $p_c(t)$, an analog system produces an output signal $q_c(t) = p_c(t-a)$, for some specified system parameter a. Assume that you are given the digital sequence $p[n] = p_c(nT)$, where sampling interval $T = 0.001s$.

(a) [1 point] In words (not equations), what does this analog system do?

(b) [2 points] Under what condition can the analog signal $p_c(t)$ be reconstructed exactly from the digital sequence $p[n]$?

(c) [3 points] Assuming that this condition is satisfied, write the equation that shows how you could exactly compute $q_c(t)$ from $p[n]$.

(d) [1 point] For a single time t, what is the computational cost of computing $q_c(t)$? (How many multiplications?)

(e) [2 points] How would you make the computation of $q_c(t)$ more efficient, while still approximately correct?

(f) [1 point] What determines the accuracy of your more efficient approximation?

Question 2 .... (14 points)

Consider two digital low-pass filter systems H_1 and H_2, where H_1 is defined by its frequency response

$$H_1(\omega) = \begin{cases} 1 & ; \quad |\omega| \leq \pi/2 \\ 0 & ; \quad |\omega| > \pi/2 \end{cases},$$

and H_2 is defined by its Z-transform

$$H_2(z) = \frac{1}{4}z + \frac{1}{2} + \frac{1}{4}z^{-1} \quad , \quad 0 < |z| < \infty.$$

(a) [4 points] For both systems, sketch amplitude and phase responses for $0 \leq \omega \leq \pi$. (Total of four sketches.)

(b) [2 points] What is the impulse response $h_1[n]$ corresponding to $H_1(\omega)$?

(c) [2 points] What is the impulse response $h_2[n]$ corresponding to $H_2(z)$?

(d) [2 points] For impulse response $h_1[n]$, write an equation for output samples $y[n]$ as a weighted sum of input samples $x[n]$.

(e) [2 points] Which low-pass filter system can be implemented more efficiently?

(f) [2 points] Which low-pass filter system is best? In what sense?

Question 3 .... (12 points)

Consider a linear time-invariant filter implemented by the following computer program:

```
float xnm2 = 0.0f,  xnm1 = 0.0f; // x[n-2] and x[n-1]
float ynm2 = 0.0f,  ynm1 = 0.0f; // y[n-2] and y[n-1]
for (int n=0; n<lxy; ++n) {
  float xn = x[n];
  float yn = y[n] = xn-xnm1+xnm2+0.9*ynm1-0.81*ynm2;
  xnm2 = xnm1;  xnm1 = xn;
  ynm2 = ynm1;  ynm1 = yn;
}
```

(a) [2 points] What is the Z-transform $H(z)$ of this filter? (Include the ROC!)

(b) [2 points] Where are the two poles?

(c) [2 points] Where are the two zeros?

(d) [2 points] Is this filter stable? Is it causal?

(e) [4 points] Sketch the amplitude response $A(\omega)$ and phase response $\phi(\omega)$.

Question 4 .... (14 points)

Consider a stable system with input $x[n]$ and output $y[n]$ related by

$$y[n+1] - \frac{5}{2}y[n] + y[n-1] = x[n].$$

(a) [2 points] What is the Z-transform $H(z)$ of this system?

(b) [2 points] Where are the two poles?

(c) [2 points] Where are the two zeros?

(d) [2 points] What is the region of convergence?

(e) [2 points] Is this system linear? Shift-invariant? Causal?

(f) [2 points] What is the impulse response $h[n]$ of this filter?

(g) [2 points] Write the loops that would implement this filter in a computer program.