# GPGN 404
# Final Exam
# December 13, 2006 Name: ____________

| Question: | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| Points: | 8 | 11 | 12 | 10 | 5 | 4 | 50 |
| Score: | | | | | | | |

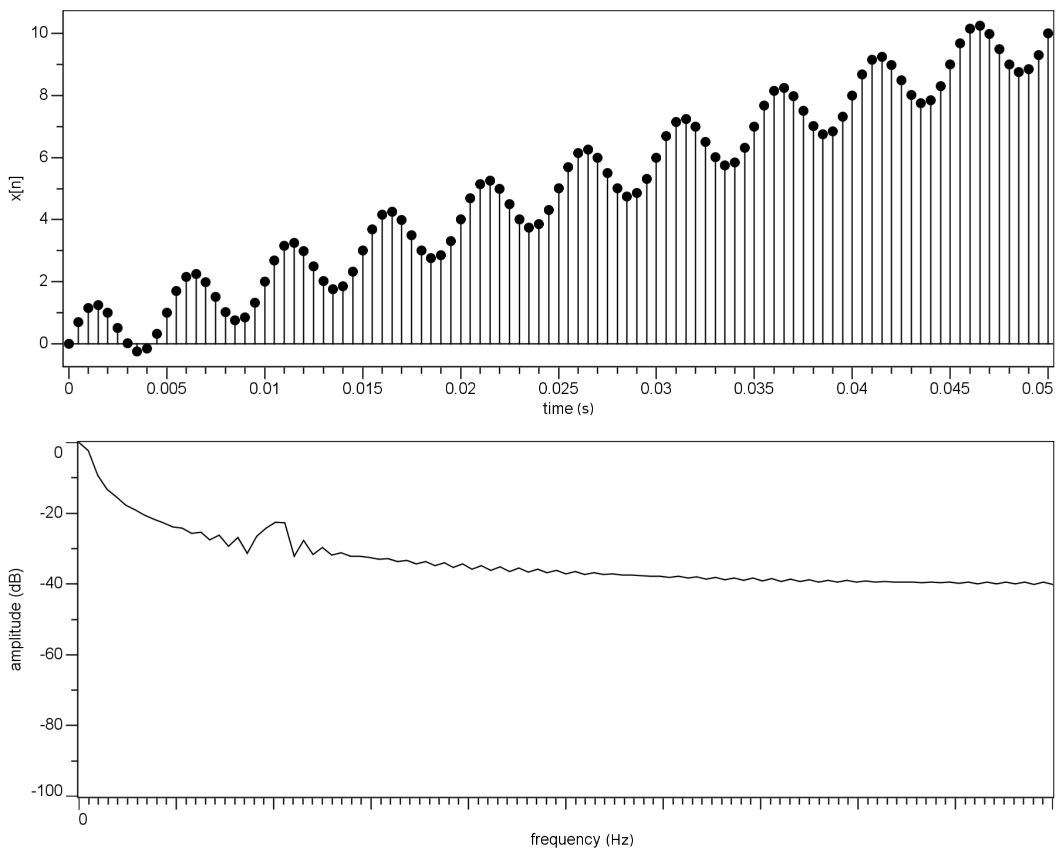


Figure 1: A sequence $x[n]$ and its amplitude spectrum. The sequence $x[n]$ is not aliased and consists of 101 samples of a continuous signal $x_c(t)$.

Question 1 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . (8 points)

(a) [2 points] What is the time sampling interval $T$, in seconds?

(b) [2 points] What is the Nyquist frequency, in Hz?

(c) [2 points] What is the frequency (in Hz) of the periodic fluctuations in $x[n]$.

(d) [2 points] Label the frequency axis below the amplitude spectrum.

Question 2 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . (11 points)

Consider the moving average filter with system function $H(z) = \frac{1}{5}(z^2 + z + 1 + z^{-1} + z^{-2})$.

(a) [2 points] Sketch the impulse response $h[n]$ of this system.

(b) [2 points] Use a well-known formula to rewrite $H(z)$ as the ratio of two polynomials in $z$.

(c) [2 points] For what two frequencies $\omega$ between 0 and $\pi$ is the amplitude response $A(\omega)$ of this filter zero?

(d) [3 points] Assume that the sequence displayed in Figure 1 is $x[n] = cn + sin(\omega_0 n)$, for some constant $c$ and frequency $\omega_0$. (You should already have determined the frequency $\omega_0$.) For this input $x[n]$, and ignoring samples near the ends, what is the output $y[n]$ of this moving average filter?

Hint: the input $x[n]$ to this LTI system is the sum of two sequences.

(e) [2 points] Write the main loop of a computer program that implements this filter.

Question 3 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . (12 points)

Design a causal time-domain notch filter to attenuate the periodic fluctations in $x[n]$ of Figure 1, as follows:

(a) [2 points] What is the frequency to be attenuated, in radians/sample?

(b) [2 points] Sketch the locations of filter poles and zeros in the complex $z$-plane.

(c) [2 points] What is the system function $H(z)$ for your filter? (Include the region of convergence.)

(d) [2 points] Modify your system function $H(z)$ so that your filter does nothing at frequency zero (DC).

(e) [2 points] Write a constant-coefficient difference equation relating filter output $y[n]$ to input $x[n]$.

(f) [2 points] Write the main loop of a computer program that implements your filter.

Question 4 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . (10 points)

Assume that you are given a discrete Fourier transform $X[k]$ of the sequence $x[n]$ of Figure 1. Also assume that a fast Fourier transform (FFT) was used, and that the number of samples after padding $x[n]$ with zeros was $N = 128$.

(a) [2 points] Why typically are only positive frequencies $\omega$ sampled in the array $X[k]$?

(b) [2 points] Assuming that only positive frequencies $\omega$ are sampled, how many values are provided in the array $X[k]$?

(c) [2 points] What is the frequency sampling interval $\Delta\omega$, in radians/sample?

(d) [2 points] What is the index $k_0$ of the sample corresponding to the frequency nearest that of the periodic fluctuations in the sequence $x[n]$?

(e) [2 points] Describe in words how you would use this index $k_0$ to implement a linear time-invariant filter that attenuates the periodic fluctuations in the sequence $x[n]$?

Question 5 ........................................................... (5 points)

Given the sequence $x[n] = x_c(nT)$ from Figure 1, ...

(a) [3 points] Implement the transformation $y_c(t) = x_c(\sqrt{t})$. That is, write an expression that exactly defines a new output sequence $y[n] = y_c(nT)$ in terms of the samples of the input sequence $x[n]$.

(b) [2 points] In practice, we often sacrifice precision for efficiency in such transformations. How would you modify your computation of $y[n]$ to make it faster, while still approximately correct?

Question 6 ........................................................... (4 points)

Give an example of $H(z)$ (system function with ROC) for a linear time-invariant system that is ...

(a) [2 points] Stable but not causal.

(b) [2 points] Causal but not stable.