GPGN 404
Final Exam
December 8, 2007 **Name:** ______________

Question:	1	2	3	4	5	Total
Points:	10	10	12	12	11	55
Score:						

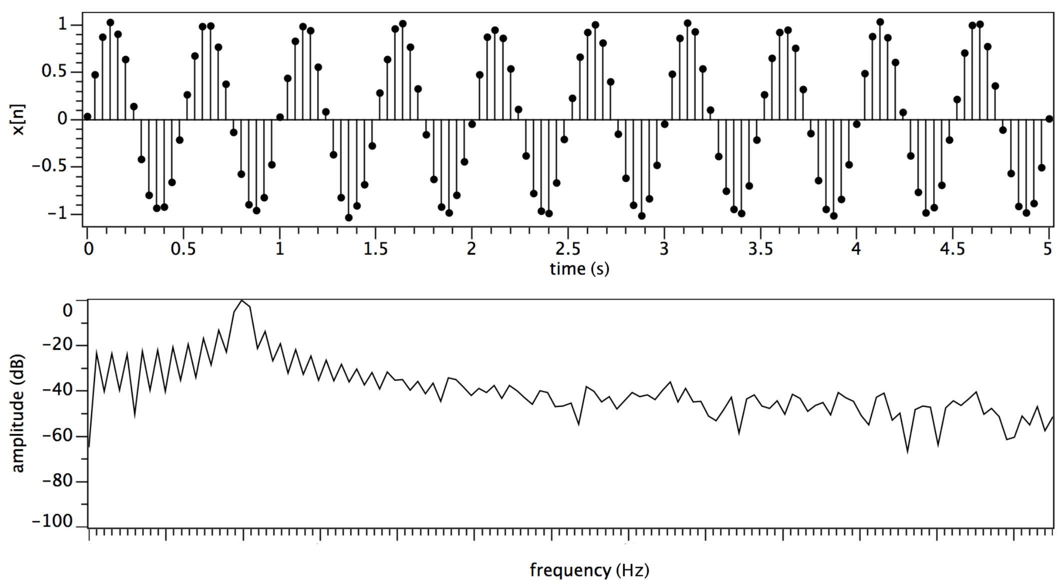

Figure 1: A sequence $x[n]$ and its amplitude spectrum. The sequence $x[n]$ is not aliased and consists of 126 samples of a continuous signal $x_c(t)$. The periodic fluctuations are noise; the interesting signal is about -40 dB down.

Question 1 .... (10 points)

(a) [2 points] What is the time sampling interval T, in seconds?

(b) [2 points] What is the Nyquist frequency, in Hz?

(c) [2 points] What is the frequency (in Hz) of the periodic fluctuations in $x[n]$.

(d) [2 points] Label the frequency axis below the amplitude spectrum.

(e) [2 points] As noted above, the periodic fluctuations are noise; the signal of interest is about -40 dB down. What does "-40 dB down" mean? Specifically, what is the ratio of signal amplitude to noise amplitude?

Question 2 .... (10 points)

Assume that you are given a discrete Fourier transform $X[k]$ of the sequence $x[n]$ of Figure 1. Assume also that a fast Fourier transform (FFT) was used, and that the number of samples after padding $x[n]$ with zeros was $N = 250$.

(a) [2 points] If only positive frequencies ω are sampled, how many complex values are provided in the array $X[k]$?

(b) [2 points] For what sample indices k are the imaginary parts of $X[k]$ zero?

(c) [2 points] What is the frequency sampling interval $\Delta\omega$, in radians/sample?

(d) [2 points] What is the frequency sampling interval ΔF, in Hz?

(e) [2 points] Imagine a simple filter that zeros amplitudes for frequencies between 1 and 3 Hz. To implement this filter, for what range of sample indices k would you zero $X[k]$?

Question 3 .... (12 points)

Design a causal notch filter to attenuate the periodic fluctations in $x[n]$ of Figure 1, as follows:

(a) [2 points] What is the frequency to be attenuated, in radians/sample?

(b) [2 points] Sketch the locations of filter poles and zeros in the complex z-plane.

(c) [2 points] What is the system function $H(z)$ for your filter? (Include the region of convergence.)

(d) [2 points] Modify your system function $H(z)$ so that your filter does nothing at frequency zero (DC).

(e) [2 points] Write a constant-coefficient difference equation relating filter output $y[n]$ to input $x[n]$.

(f) [2 points] Write the main loop of a computer program that implements your filter.

Question 4 .... (12 points)

Consider an LTI system with the following frequency response

$$H(\omega) = \begin{cases} 1, & \text{if } |\omega| \leq \pi/2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) [2 points] Sketch this frequency response $H(\omega)$ for frequencies ω in the range $-\pi \leq \omega \leq \pi$.

(b) [4 points] What is the impulse response $h[n]$ of this system?

(c) [2 points] Sketch the impulse response $h[n]$ of this system.

(d) [2 points] Suppose the sequence $x[n]$ of Figure 1 is input to this system to obtain an output sequence $y[n]$. Using the amplitude spectrum in Figure 1 as a guide, sketch the amplitude spectrum of the output sequence $y[n]$.

(e) [2 points] Such a system might be used prior to subsampling the sequence $y[n]$. Specifically, we might use it before computing $z[n] = y[2n]$. Why?

Question 5 .... (11 points)

Consider three moving-average filters with system functions:

$$\begin{aligned} H_1(z) &= \frac{1}{3}(1 + z^{-1} + z^{-2}) \\ H_2(z) &= \frac{1}{3}(z^2 + z + 1) \\ H_3(z) &= H_1(z)\, H_2(z) \end{aligned}$$

(a) [4 points] Sketch the impulse responses of all three systems.

(b) [1 point] For what frequency ω between 0 and π is the amplitude response $A_1(\omega)$ of the filter H_1 zero?

(c) [1 point] For what frequency ω between 0 and π is the amplitude response $A_2(\omega)$ of the filter H_2 zero?

(d) [1 point] For what frequency ω between 0 and π is the amplitude response $A_3(\omega)$ of the filter H_3 zero?

(e) [2 points] Give two reasons why the filter H_3 is a better moving-average filter than either H_1 or H_2.

(f) [2 points] Which of these filters are causal? Which are stable?