GPGN 404
Final Exam
December 10, 2008 **Name:** ____________

Question:	1	2	3	4	5	Total
Points:	8	10	12	8	12	50
Score:						

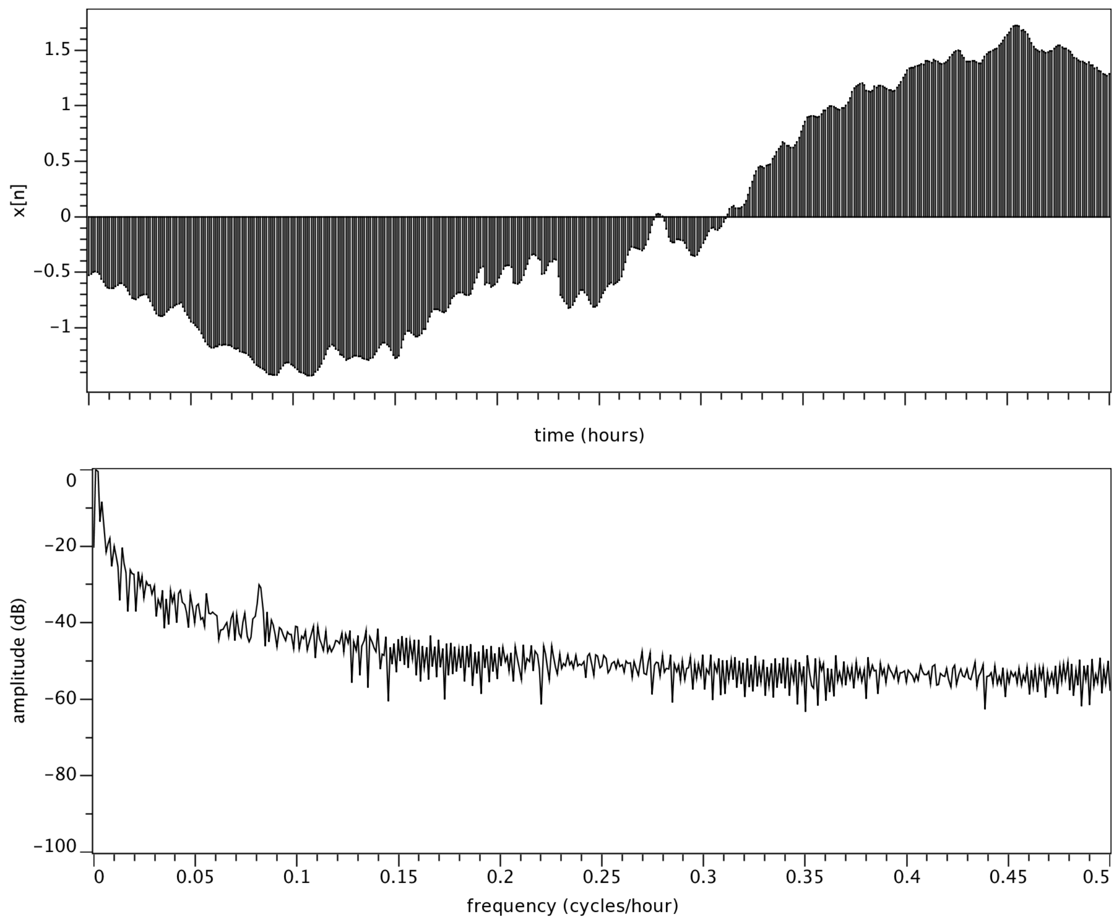

Figure 1: The sequence $x[n]$ consists of 501 samples, is not aliased, and the Nyquist frequency is 0.5 cycles/hour. Fluctuations with different frequencies are apparent. The frequency 1/12 (≈ 0.083) cycles/hour corresponds to Earth-Moon tidal fluctuations that here are considered noise.

Question 1 . . . . (8 points)

(a) [2 points] What is the time sampling interval T, in hours?

(b) [2 points] Label the time axis.

(c) [2 points] Annotate features in both plots to highlight (1) the tidal noise and (2) the most significant signal apparent at a very low (but non-zero) frequency. Indicate corresponding features in both time and frequency domains.

(d) [2 points] Roughly, what is the amplitude ratio (a fraction) between the tidal noise and the low-frequency signal?

Question 2 . . . . (10 points)

Consider the task of attenuating the tidal noise in Figure 1 with a frequency-domain filter. Assume that you will compute a fast Fourier transform $X[k]$ of the sequence $x[n]$ and that the FFT length is $N = 1000$.

(a) [2 points] Give two reasons why you would use an FFT length N that is greater than (not equal to) the number of samples (501) in the sequence $x[n]$.

(b) [2 points] What values would you assign to the extra samples before the FFT? (If helpful to explain your answer, draw a picture.) Why?

(c) [2 points] What is the frequency sampling interval $\Delta\omega$, in radians/sample?

(d) [2 points] What is the frequency sampling interval ΔF, in cycles/hour?

(e) [2 points] Determine the index k_0 of the sample in $X[k]$ that contains most of the tidal noise.

Question 3 . . . . (12 points)

Design a causal notch filter to attenuate the tidal noise in $x[n]$ of Figure 1, as follows:

(a) [2 points] What is the frequency to be attenuated, in radians/sample?

(b) [2 points] Sketch the locations of filter poles and zeros in the complex z-plane.

(c) [2 points] What is the system function $H(z)$ for your filter? (Include the region of convergence.)

(d) [2 points] Modify your system function $H(z)$ so that your filter does nothing at frequency zero (DC).

(e) [2 points] Write a constant-coefficient difference equation relating filter output $y[n]$ to input $x[n]$.

(f) [2 points] Write the main loop of a computer program that implements your filter.

Question 4 . . . . (8 points)

Consider a digital implementation of the transformation $y_c(t) = x_c(t - s)$, where s denotes a system parameter. Given the sequence $x[n] = x_c(nT)$ from Figure 1, ...

(a) [1 point] In simple terms (no equations), what does this system do?

(b) [3 points] Write an expression that defines exactly a system with an output sequence $y[n] = y_c(nT)$ in terms of the samples of the input sequence $x[n]$, sampling interval T, and system parameter s.

(c) [2 points] In practice, how might you approximate this exact ideal system to make it more efficient?

(d) [2 points] Using Figure 1 as a guide, sketch the amplitude spectrum of the output $y[n]$ for system parameter $s = 1.5$ hours.

Question 5 .... (12 points)

Consider a digital system with parameter m and z-transform:

$$H(z) = \frac{1}{m}(1 + z^{-1} + z^{-2} + \cdots + z^{-m+1}).$$

(a) [2 points] What is the region of convergence of $H(z)$? Is this system stable? Is it causal?

(b) For $m = 4$, ...

i. [2 points] Sketch the impulse response of this system.

ii. [2 points] For input $x[n] = cos(\pi n)$, what is the output $y[n]$?

(c) For larger m (much greater than 4), convolving with the impulse response of this filter is costly. For a more efficient implementation, ...

i. [2 points] Rewrite $H(z)$ as the ratio of two polynomials.

ii. [2 points] Write a linear constant-coefficient difference equation that relates system input $x[n]$ and output $y[n]$.

iii. [2 points] Write the main loop of a computer program that computes an output array $y[n]$ from an input array $x[n]$.