GPGN 404
Final Exam
December 16, 2009

Name: ______________________

Question:	1	2	3	4	Total
Points:	8	12	18	12	50
Score:					

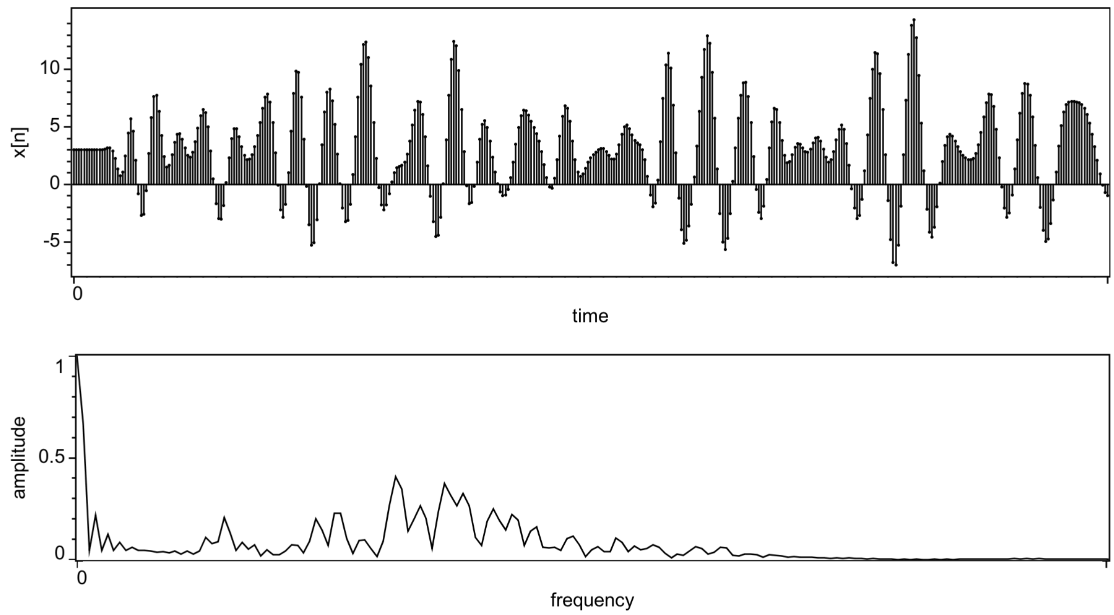

Figure 1: The sequence $x[n]$ consists of $N = 401$ samples, where the sampling interval is $T = 2$ ms and the time of first sample is zero. The amplitude spectrum has been normalized so that the amplitude at zero Hz is one.

Question 1 .... (8 points)

(a) [2 points] What is the Nyquist frequency, in Hz (cycles per second)?

(b) [2 points] Label the time axis, with units of seconds.

(c) [2 points] In the amplitude spectrum, the minimum frequency plotted is zero. *The maximum frequency plotted is not the Nyquist frequency.* Label the frequency axis, with units of Hz.

(d) [2 points] What attribute of the sequence $x[n]$ best explains the large peak in the amplitude spectrum at zero frequency?

Question 2 .... (12 points)

For the sequence $x[n]$ in Figure 1, assume that anything above 50 Hz is noise, and consider the task of attenuating this high-frequency noise with a frequency-domain filter. The sequence $x[n]$ contains 401 samples, and you choose an FFT length $N = 2000$ samples.

(a) [2 points] Explain why a smaller FFT length $N = 500$ might not be adequate.

(b) [2 points] Explain why you *cannot* choose an FFT length $N = 401$. (Hint: the number 401 is prime.)

(c) [2 points] After the FFT, the values $X[k]$ are generally complex, with real and imaginary parts. For which two indices k are the imaginary parts guaranteed to be zero?

(d) [2 points] What is the frequency sampling interval ΔF, in Hz?

(e) [4 points] To attenuate the high-frequency noise above 50 Hz, for what range of indices k would you zero $X[k]$?

Question 3 .... (18 points)

Refer to the sequence $x[n]$ and amplitude spectrum in Figure 1. Note the large peak in the amplitude spectrum at frequency $F = 0$ Hz.

(a) [2 points] Specify the system response $H(z)$ for a causal system, with exactly one pole and one zero, that will zero the amplitude at $F = 0$ Hz. Place the one pole for your system at $z = 0$.

(b) [2 points] Sketch the impulse response $h[n]$ of your filter.

(c) [2 points] Express the output $y[n]$ of your system in terms of the input $x[n]$.

(d) [4 points] Sketch the amplitude and phase responses $A(\omega)$ and $\phi(\omega)$ of your system for $-\pi < \omega < \pi$. (Units of ω are radians per sample.)

(e) [2 points] What is the amplitude response of your filter for frequency $F = 50$ Hz? (Express your answer in terms of a trigonometric function.)

(f) [2 points] Move the pole of your filter so that the amplitude response is nearly one for non-zero frequencies. Specify your modified system response $H(z)$.

(g) [2 points] Now include a scale factor so that the amplitude response is exactly one at the Nyquist frequency. Specify your modified system response $H(z)$.

(h) [2 points] Express the output $y[n]$ of your modified system in terms of the input $x[n]$.

Question 4 . . . . (12 points)

Consider two resampling systems that compute output sequences defined by $y_1[n] = x[2n]$ and $y_2[n] = x[4n]$ for the input sequence $x[n]$ displayed in Figure 1. (Recall that the sampling interval for $x[n]$ is $T = 2$ ms.)

(a) [2 points] What are the sampling intervals T_1 and T_2 for the two outputs?

(b) [2 points] For the frequency range shown in Figure 1, sketch (roughly) the amplitude spectra $A_1(F)$ and $A_2(F)$ for the output sequences $y_1[n]$ and $y_2[n]$.

(c) [2 points] Is the sequence $y_1[n]$ aliased? Why or why not?

(d) [2 points] Is the sequence $y_2[n]$ aliased? Why or why not?

(e) [4 points] Write an analytical expression for a third resampling system with output $y_3[n]$ that has sampling interval $T_3 = 1$ ms, where the input is again the sequence $x[n]$ of Figure 1.