GPGN 404
Final Exam
December 16, 2010 **Name:** ____________________

Question:	1	2	3	4	Total
Points:	8	10	22	10	50
Score:					

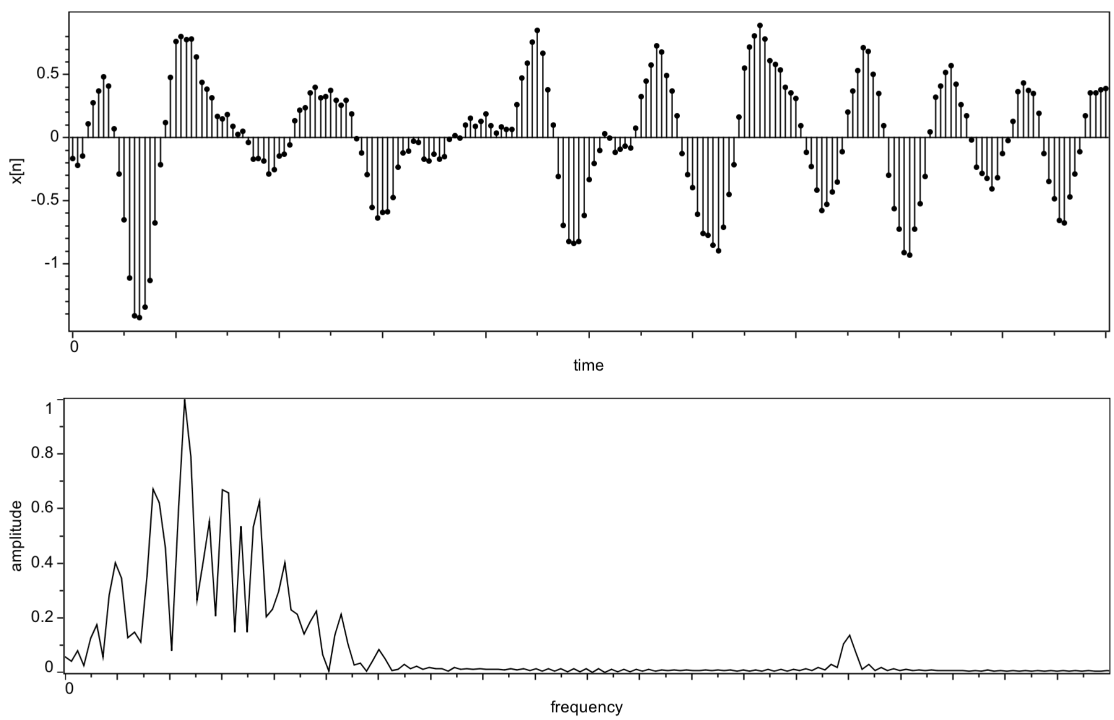

Figure 1: The sequence $x[n]$ consists of $N = 201$ samples, where the sampling interval is $T = 1$ ms and the time of first sample is zero.

Question 1 . . . . (8 points)

(a) [2 points] What is the Nyquist frequency, in Hz (cycles per second)?

(b) [2 points] Label the time axis, with units of seconds.

(c) [2 points] In the amplitude spectrum, the minimum frequency plotted is zero. *The maximum frequency plotted is not the Nyquist frequency.* Label the frequency axis, with units of Hz.

(d) [2 points] The sequence $x[n]$ is contaminated with high-frequency noise with relatively low amplitude. At what frequency (in Hz) is this noise apparent in the amplitude spectrum?

Question 2 .... (10 points)

For the sequence $x[n]$ in Figure 1, consider the task of attenuating the high-frequency noise with a frequency-domain filter. The sequence $x[n]$ contains 201 samples, and you choose an FFT length $N = 400$ samples.

(a) [2 points] $201 = 3 \times 67$. Explain why you would not choose an FFT length $N = 201$.

(b) [2 points] $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$. Explain why an FFT length $N = 216$ might be too small.

(c) [2 points] After the FFT, the values $X[k]$ are generally complex, with real and imaginary parts. For which two indices k are the imaginary parts guaranteed to be zero?

(d) [2 points] What is the frequency sampling interval ΔF, in Hz?

(e) [2 points] Determine the index k_0 of the sample in $X[k]$ that contains most of the noise.

Question 3 .... (22 points)

Refer to the sequence $x[n]$ and amplitude spectrum in Figure 1. Design a causal notch filter to zero anything at the noise frequency, while preserving signal at other frequencies.

(a) [4 points] Sketch the locations of the poles and zeros for your filter.

(b) [4 points] Sketch the amplitude and phase responses $A(\omega)$ and $\phi(\omega)$ of your system for $-\pi < \omega < \pi$. (Units of ω are radians per sample.)

(c) [4 points] Specify the system response $H(z)$ of your filter. Include a scale factor to ensure that the zero-frequency (DC) response is one. Define all coefficients (b_0, b_1, ...) in $H(z)$, but assume that a computer program will be used to compute their numerical values.

(d) [2 points] Write a linear constant-coefficient difference equation for the output $y[n]$ of your system in terms of the input $x[n]$.

(e) [2 points] Now suppose that you apply your notch filter in the opposite direction so that the system is *anti-causal*. Again express the output $y[n]$ in terms of the input $x[n]$.

(f) [2 points] Specify the system response $H(z)$ of the anti-causal filter.

(g) [2 points] Sketch the amplitude and phase responses $A(\omega)$ and $\phi(\omega)$ of the anti-causal filter, for $-\pi < \omega < \pi$.

(h) [2 points] Let $y_1[n]$ denote the output of the causal system and $y_2[n]$ the output of the anti-causal system. We can apply both filters and average the two outputs to obtain the composite output $y[n] = (y_1[n] + y_2[n])/2$. Sketch the amplitude and phase responses $A(\omega)$ and $\phi(\omega)$ of the composite system, for $-\pi < \omega < \pi$.

Question 4 . . . . (10 points)

Consider resampling the sequence $x[n]$ of Figure 1, without anti-alias filtering.

(a) [4 points] Specify a resampling system for output $y_1[n]$ with sampling interval 2 ms. Express the output $y_1[n]$ in terms of the input $x[n]$.

(b) [2 points] At what frequency less than 250 Hz will aliased high-frequency noise be apparent in the output $y_1[n]$.

(c) [4 points] Specify a resampling system for output $y_2[n]$ with sampling interval 0.1 ms. Express the output $y_2[n]$ in terms of the input $x[n]$.