GPGN 404
Final Exam
December 13, 2011 **Name:** _____

Question:	1	2	3	4	Total
Points:	8	8	4	30	50
Score:					

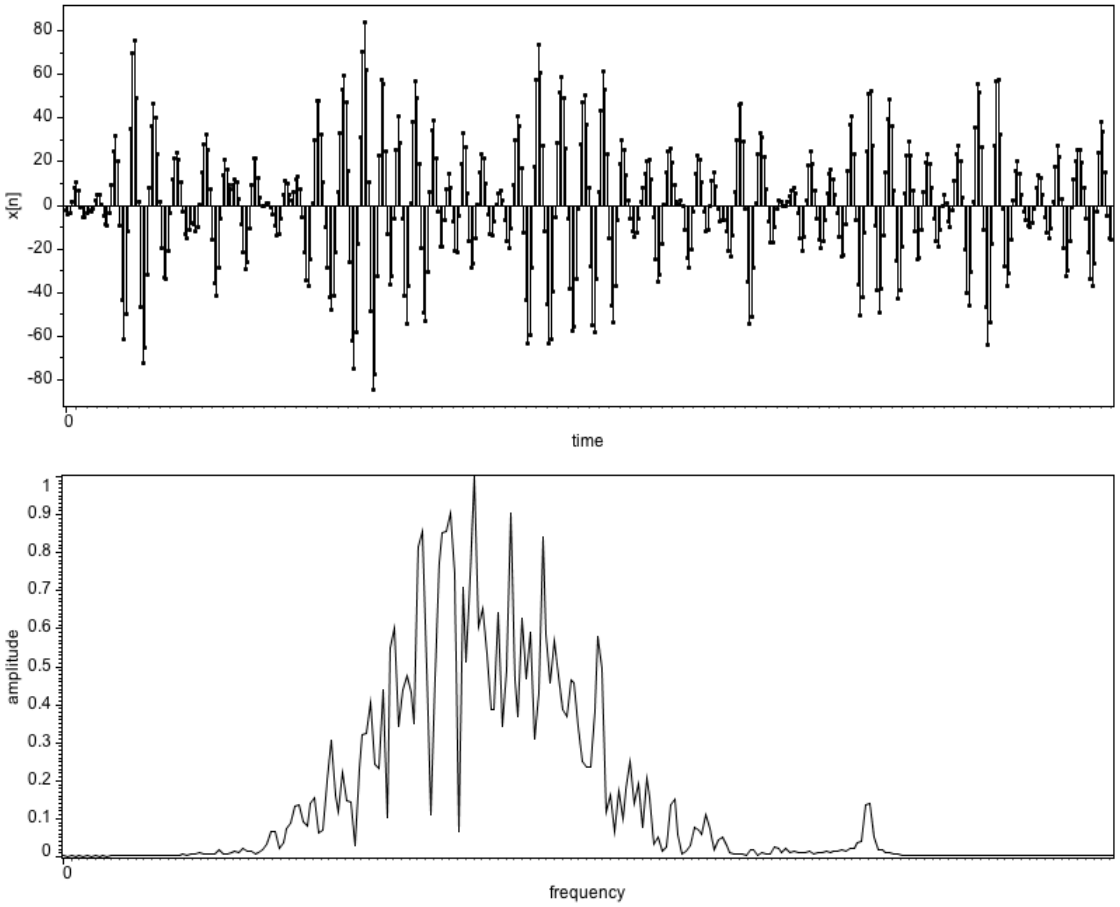

Figure 1: The sequence $x[n]$ consists of $N = 501$ samples, where the sampling interval is $T = 0.002$ s and the time of first sample is zero. The small high-frequency peak corresponds to weak but important signal that is almost invisible in $x[n]$.

Question 1 ..... (8 points)

(a) What is the Nyquist frequency, in Hz (cycles per second)?

(b) Label the time axis, with units of seconds.

(c) In the amplitude spectrum, the minimum frequency plotted is zero. *The maximum frequency plotted is not the Nyquist frequency.* Label the frequency axis, with units of Hz.

(d) At what frequency (in Hz) is the weak high-frequency signal apparent in the amplitude spectrum?

Question 2 ..... (8 points)

Consider the sequence $x[n]$ in Figure 1. Describe in words and equations (not a computer program) how you would use a fast Fourier transform (FFT) to implement a system that highlights the high-frequency signal, by attenuating other frequencies. Be specific enough that a computer programmer could implement your system in any programming language. Specifically, ...

(a) How would you choose the length for the FFT?

(b) How would you construct from $x[n]$ an array of values to be transformed?

(c) Assume that you have performed the FFT to obtain $X[k]$, where k is the frequency sampling index. How would you obtain the array $Y[k]$ from $X[k]$?

(d) Finally, how would you obtain the output sequence $y[n]$?

Question 3 ..... (4 points)

Suppose that you are given a notch filter that can zero amplitude at any specified frequency.

(a) Describe how you might use this notch filter differently, to preserve the high-frequency signal in Figure 1, while attenuating *all other* frequencies.

(b) How might this system be better or worse than the one based on FFTs?

Question 4 ..... (30 points)

Consider a linear time-invariant system with response $H(z) = \frac{1}{2}z + 1 + \frac{1}{2}z^{-1}$.

(a) Sketch the impulse response for this system.

(b) Is this system causal? Is it stable? For both questions, explain your answers.

(c) What is the frequency response $H(\omega)$ for this system?

(d) Sketch the amplitude and phase responses $A(\omega)$ and $\phi(\omega)$ for this system for $-\pi < \omega < \pi$. Label axes carefully.

(e) Where are the poles and zeros for this system?

(f) What is the region of convergence for the system response $H(z)$?

(g) In plain simple English (not equations), what does this system do?

(h) Write a computer program fragment that for this system computes an output array of N_t samples $y[n]$ from an input array of N_t samples $x[n]$, for $n = 0, 1, 2, \ldots, N_t - 1$.

(i) If input $x[n] = 7$ (is constant), what is the output $y[n]$?

(j) For input $x[n] = \cos(\pi n/2)$, what is the output $y[n]$?

(k) Let $X(\omega)$ and $Y(\omega)$ denote discrete-time Fourier transforms (DTFTs) of arbitrary input and output sequences $x[n]$ and $y[n]$. For this system, write $Y(\omega)$ in terms of $X(\omega)$.

(l) Let $X[k])$ and $Y[k]$ denote discrete Fourier transforms (DFTs) of arbitrary input and output sequences $x[n]$ and $y[n]$. Assume that the input sequence $x[n]$ has length N, before transforming. For this system, write $Y[k]$ in terms of $X[k]$.

(m) If the time sampling interval $T = 0.2$ s, what frequency (in Hz) is most attenuated by this system?