GPGN 404
Final Exam
December 10, 2012 **Name:** _____

Question:	1	2	3	4	5	Total
Points:	8	8	6	20	8	50
Score:						

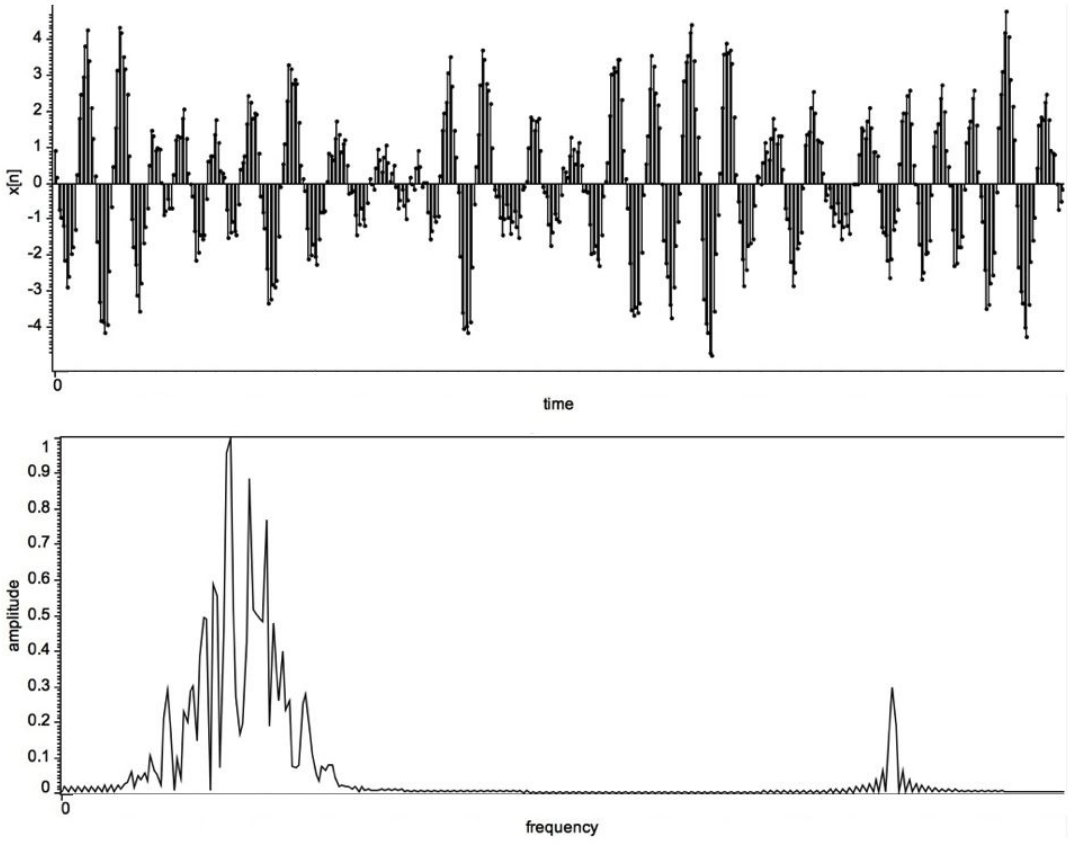

Figure 1: The sequence $x[n]$ consists of $N = 501$ samples, where the sampling interval is $T = 0.001$ s and the time of first sample is zero. The high-frequency peak corresponds to noise that we wish to attenuate.

Question 1 ..... (8 points)

(a) Label the time axis, with units of seconds.

(b) What is the Nyquist frequency, in Hz (cycles per second)?

(c) In the amplitude spectrum, the minimum frequency plotted is zero. *The maximum frequency at the far right of the frequency axis is not the Nyquist frequency.* Label the frequency axis, with units of Hz.

(d) At what frequency (in Hz) is the high-frequency noise apparent in the amplitude spectrum?

Question 2 ..... (8 points)

Consider frequency-domain filtering to attenuate the high-frequency noise in the sequence $x[n]$ of Figure 1. Assume that we will use length $N = 1008 = 16 \times 9 \times 7$ for the FFT.

(a) Why not instead use the prime number $N = 1009$ for the FFT?

(b) Why not instead use the number $N = 504 = 9 \times 8 \times 7$ for the FFT?

(c) The sequence $x[n]$ is given for only $n = 0, 1, \ldots, 500$. Before FFT, what values should we put in the array for indices $n = 501, 502, \ldots, 1007$?

(d) Assume that we have performed the FFT to obtain complex numbers $X[k]$, where k is the frequency sampling index. What value of k most nearly corresponds to the frequency of the high-frequency noise?

Question 3 ..... (6 points)

Consider the two-sided exponential smoothing filter with symmetric impulse response $h[n] = \left(\frac{1}{3}\right)^{|n|}$. For this filter, specify the corresponding system response $H(z)$, with region of convergence, and the locations of all poles and zeros.

Question 4 ..... (20 points)

Consider a linear time-invariant system with response $H(z) = 1 - z^{-2}$.

(a) Sketch the impulse response for this system.

(b) Is this system causal? Is it stable? For both questions, explain your answers.

(c) What is the frequency response $H(\omega)$ for this system?

(d) Sketch the amplitude and phase responses $A(\omega)$ and $\phi(\omega)$ for this system for $-\pi < \omega < \pi$. Label axes carefully.

(e) Where are the poles and zeros for this system?

(f) What is the region of convergence for the system response $H(z)$?

(g) If input $x[n] = 3$ (is constant for all n), what is the output $y[n]$?

(h) Write a computer program fragment that for this system computes an output array of N_t samples $y[n]$ from an input array of N_t samples $x[n]$, for $n = 0, 1, 2, \ldots, N_t - 1$.

Question 5 ..... (8 points)

Consider the system $y[n] = x[4n]$ and its application to the sequence $x[n]$ of Figure 1.

(a) Prove that this system is linear.

(b) Prove that this system is *not* time-invariant.

(c) For the output sequence $y[n]$, what is the Nyquist frequency (in Hz)?

(d) In the output sequence $y[n]$, at what low frequency (less than 30 Hz) will the aliased high-frequency noise be apparent?