GPGN 404
Final Exam
December 12, 2013 **Name:** _____

Question:	1	2	3	4	Total
Points:	8	8	10	24	50
Score:					

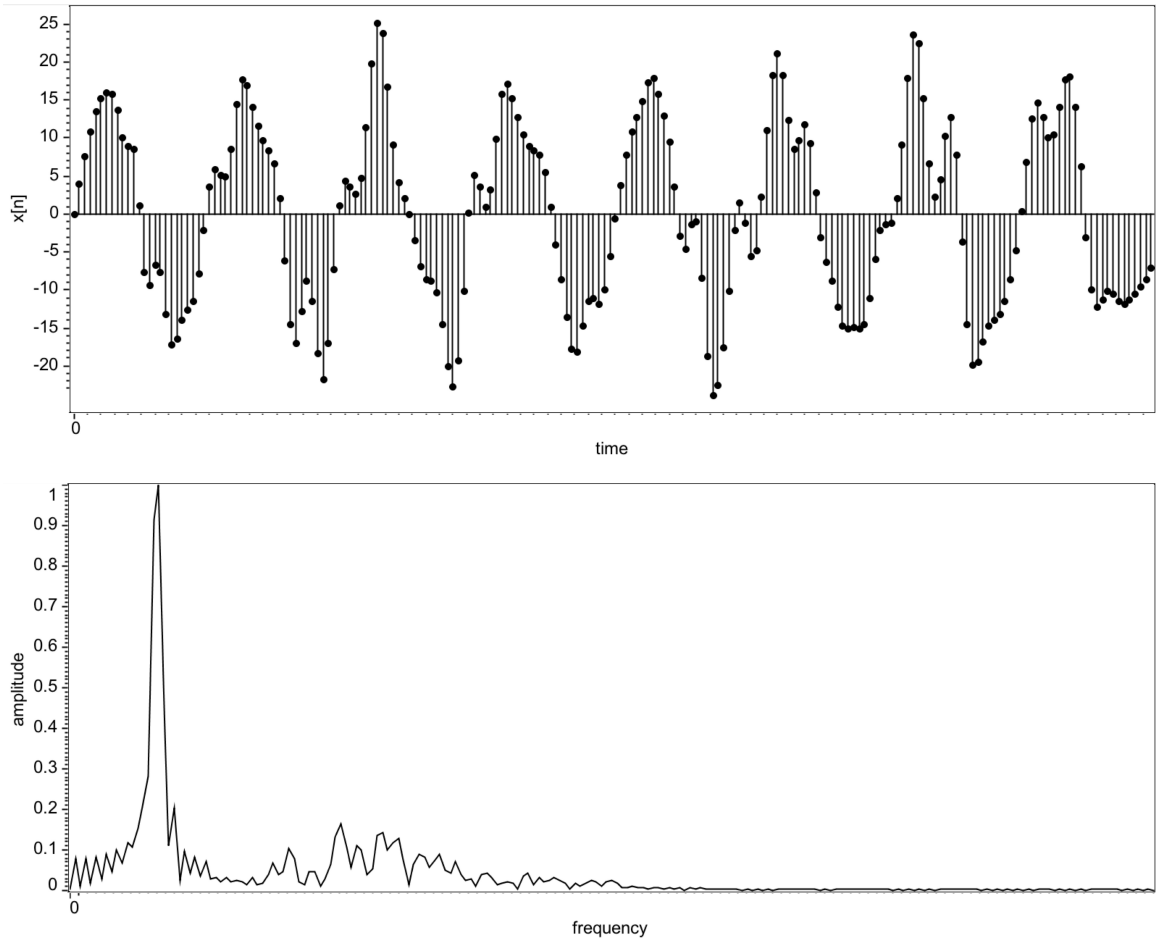

Figure 1: The sequence $x[n]$ consists of $N = 200$ samples, where the sampling interval is $T = 0.04$ s and the time of first sample is zero. The low-frequency peak corresponds to signal that we wish to preserve, while attenuating noise apparent at higher frequencies. Let's assume that $x[n]$ is not aliased.

Question 1 ..... (8 points)

(a) Label the time axis, with units of seconds.

(b) What is the Nyquist frequency, in Hz (cycles per second)?

(c) At what frequency (in Hz) is the low-frequency signal most apparent in the amplitude spectrum?

(d) Label the frequency axis, with units of Hz.

Question 2 ..... (8 points)

Consider frequency-domain filtering to implement a low-pass filter. Our filter will simply zero the discrete Fourier transform $X[k]$ of the sequence $x[n]$ shown in Figure 1, for all frequencies above a specified cutoff frequency. Let's choose the cutoff frequency to lie safely between the low-frequency signal and the noise apparent at higher frequencies.

(a) In Figure 1, label the following frequencies:

- zero frequency
- Nyquist frequency
- peak frequency of the signal
- cutoff frequency for our filter

(b) List two reasons why $N = 432 = 9 * 16$ is a good choice for FFT length.

(c) In attenuating the noise at higher frequencies, for what range of indices k would you zero the complex numbers $X[k]$ in a simple low-pass filter.

Question 3 ..... (10 points)

Given the sequence $x[n]$ in Figure 1, sketch (roughly, but label axes) the amplitude spectrum for sequences $y[n]$ and $z[n]$ defined by:

(a) $y[n] = 2x[n]$

(b) $z[n] = x[2n]$

(c) Explain why the sequence $y[n]$ is, or is not, aliased.

(d) Explain why the sequence $z[n]$ is, or is not, aliased.

Question 4 ..... (24 points)

Consider an LTI system with the following system response:

$$H(z) = \frac{1 - z^{-8}}{1 - z^{-1}}, \quad |z| > 0$$

(a) What is the impulse response $h[n]$ for this system?

(b) Sketch locations of all poles and zeros for this system.

(c) Sketch (roughly, with labeled axes, but without deriving) the amplitude response $A(\omega)$ for this system, for $-\pi \leq \omega \leq \pi$.

(d) In terms of an input sequence $x[n]$ and output sequence $y[n]$, write a linear constant-coefficient difference equation for this system.

(e) If we initialize the output sequence using $y[0] = x[0]$, what values have we implicitly assumed for $x[-1],\ x[-2],\ x[-3],\ ...$?

(f) Given an array of input samples $x[n]$ for $n = 0, 1, 2, \ldots, N-1$, write two computer program fragments — two simple loops will do — to compute output samples $y[n]$ for

1. $n = 1, 2, \ldots, 7$:

2. $n = 8, 9, \ldots, N-1$:

(g) Explain in words, not equations, how you would implement a system with the following system response:

$$H(z)H(z^{-1}), \quad 0 < |z| < \infty.$$