GPGN 404
1st Midterm Exam
September 29, 2006 **Name:** ____________________

Question:	1	2	3	4	5	Total
Points:	5	5	4	18	18	50
Score:						

Question 1 . . . . (5 points)

Let $h[n] = \delta[n+1] - 2\delta[n] + \delta[n-1]$, where $\delta[n]$ denotes the unit-impulse sequence. Let $x[n] = u[n]$, where $u[n]$ denotes the unit-step sequence. Sketch the sequences (a) $h[n]$, (b) $x[n]$, and (c) $y[n] = h[n] * x[n]$, where $*$ denotes convolution. (Label axes.)

Question 2 . . . . (5 points)

Someone wants you to pay $10,000 for their "black-box" digital system, which takes an input sequence $x[n]$ and produces an output sequence $y[n]$. They will not show you the computer program that implements their system, but they assure you that it is really great.

Being a clever Mines student, you feed a unit-impulse to this system and record the impulse response $h[n]$. Can you now implement the system yourself? In other words, can you compute the output $y[n]$ of this system for any input $x[n]$?

If so, how would you do it? If not, why not?

Question 3 . . . . (4 points)

Given the convolution sum $y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$, and using the definition of the discrete-time Fourier transform, prove the convolution theorem; i.e., that $Y(\omega) = H(\omega)X(\omega)$.

Question 4 .... (18 points)

Consider the digital system defined by $y[n] = \frac{1}{3}(x[n+1] + x[n] + x[n-1])$.

(a) [3 points] Is this system linear? Time-invariant? Causal?

(b) [3 points] Sketch the impulse response $h[n]$ of this system. (Label axes.)

(c) [4 points] What is the frequency response $H(\omega)$ of this system?

(d) [4 points] Using the frequency response $H(\omega)$, show that the output $y[n]$ of this system for input $x[n] = cos(\pi n)$ is $y[n] = c \times cos(\pi n)$. What is the constant c?

(e) [4 points] Assume a bounded input sequence $x[n]$ such that $|x[n]| < 1$ for all n. For such an input sequence, how is the output sequence $y[n]$ bounded?

Question 5 .... (18 points)

Consider a causal system described by the constant-coefficient difference equation $y[n] + \frac{1}{2}y[n-1] = 2x[n-1]$.

(a) [2 points] Is this system linear? Time-invariant?

(b) [4 points] Sketch the impulse response $h[n]$ for this system. (Label axes.)

(c) [4 points] What is the frequency response $H(\omega)$ of this system?

(d) [4 points] Assume a bounded input sequence $x[n]$ such that $|x[n]| < 1$ for all n. For such an input sequence, how is the output sequence $y[n]$ bounded?

(e) [4 points] Write computer code to compute $y[n]$ for $n = 0, 1, 2, ..., N-1$, given input $x[n]$ for $n = 0, 1, 2, ..., N-1$.