GPGN 404
1st Midterm Exam
October 5, 2007 **Name:** _____

Question:	1	2	3	4	5	6	Total
Points:	5	5	5	5	18	22	60
Score:							

Question 1 ..... (5 points)

Let $u[n]$ denote the unit-step sequence. Sketch (with labeled axes) the following sequences $x[n]$:

(a) $x[n] = -u[n]$

(b) $x[n] = u[-n]$

(c) $x[n] = u[-n-1]$

(d) $x[n] = u[n+2] - u[n-3]$

Question 2 ..... (5 points)

Systems for processing digital signals can be characterized by several fundamental properties. What two of those properties best characterize the system $y[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$?

Question 3 ..... (5 points)

Prove that convolution is commutative. In other words, if $y[n] = h[n] * x[n]$, then $y[n] = x[n] * h[n]$.

Question 4 ..... (5 points)

Consider a periodic sequence $x[n]$ obtained by sampling a sinusoidal signal $x(t) = \sin(2\pi Ft)$ with sampling interval $T = 0.002$ s. The period of $x[n]$ is 10 samples. Specify two positive frequencies F (in Hz) that would yield the same periodic sequence $x[n]$.

Question 5 ..... (18 points)

A causal stable linear time-invariant system has frequency response

$$H(\omega) = \frac{e^{j\omega}}{1 - \frac{1}{2}e^{-j\omega 2}}.$$

(a) [2 points] What is the DC (zero-frequency) response?

(b) [3 points] Write the difference equation for this system.

(c) [4 points] Sketch the impulse response $h[n]$. (Label axes.)

(d) [4 points] Stability implies that bounded input yields bounded output. Assume that an input sequence $x[n]$ is bounded such that $|x[n]| < 2$. ($B_x = 2$.) What is the bound B_y on the output sequence $y[n]$?

Question 6 ..... (22 points)

Consider a digital linear time-invariant system that approximates a derivative $y(t) = x'(t)$. More than one such system is possible. Pick a simple one, and then ...

(a) [3 points] Write an expression for the output sequence $y[n]$ of this system in terms of the input sequence $x[n]$ and the sampling interval T.

(b) [3 points] Sketch the impulse response $h[n]$ for this system. (Label axes.)

(c) [3 points] What is the frequency response $H(\omega)$ of this system?

(d) [2 points] What is the DC (zero-frequency) response of this system?

(e) [3 points] Given the output $y_1[n]$ for some input sequence $x_1[n]$, how could you most quickly determine the output $y_2[n]$ for the shifted input sequence $x_2[n] = x_1[n + 3]$?

(f) [4 points] What is the *real-valued* output $y[n]$ for input $x[n] = \cos(\pi n/2)$? (Write your answer in terms of cos and/or sin functions, with no complex numbers.)

(g) [4 points] Write computer code (in any programming language you know) to compute $y[n]$ for $n = 0, 1, 2, ..., N-1$, given $x[n]$ for $n = 0, 1, 2, ..., N-1$. Assume that $x[n] = 0$ for $n < 0$ and $n \geq N$.