GPGN 404
1st Midterm Exam
October 2, 2009 **Name:** ____________________

Question:	1	2	3	4	5	6	Total
Points:	8	5	5	6	8	18	50
Score:							

Question 1 . . . . (8 points)

Consider the sequence $x[n] = \sin(2\pi f n)$.

(a) [4 points] If $x[n]$ is periodic with period $N = 20$ samples, what is the frequency f (in cycles per sample)?

(b) [2 points] Specify a second frequency f that yields the same sequence $x[n]$.

(c) [2 points] Specify a frequency f for which the sequence $x[n]$ is *not* periodic.

Question 2 . . . . (5 points)

Let $h[n] = 2\delta[n+1] - \delta[n] - \delta[n-1]$, where $\delta[n]$ denotes the unit-impulse sequence. Let $x[n] = u[n] - u[n-3]$, where $u[n]$ denotes the unit-step sequence. Sketch the sequences (a) $h[n]$, (b) $x[n]$, and (c) $y[n] = h[n] * x[n]$, where $*$ denotes convolution. (Label axes.)

Question 3 . . . . (5 points)

Given *only* the impulse response $h[n]$ of an LTI system, how can you

(a) [3 points] compute the output $y[n]$ for any input $x[n]$?

(b) [2 points] determine whether the system is stable?

Question 4 .... (6 points)

Given that the discrete-time Fourier transform (DTFT) of $x[n]$ is $X(\omega)$, prove that

(a) [3 points] the DTFT of $x[n+3]$ is $X(\omega)e^{j\omega 3}$.

(b) [3 points] the DTFT of $x[-n]$ is $X(-\omega)$.

Question 5 .... (8 points)

(a) [4 points] Describe in words (not a computer program) a non-linear system $y[n] = T\{x[n]\}$ that removes isolated noise spikes from any sequence $x[n]$.

(b) [4 points] For inputs $x_1[n] = \delta[n]$, $x_2[n] = u[n-1]$, and $x[n] = x_1[n] + x_2[n]$, sketch the corresponding outputs $y_1[n]$, $y_2[n]$, and $y[n]$ for your system, and thereby *prove* that your system is non-linear.

Question 6 . . . . (18 points)

Consider a stable system described by the constant-coefficient difference equation $y[n] - \frac{1}{4}y[n-1] = 2\,x[n-2]$.

(a) [3 points] Is this system linear? Time-invariant? Causal?

(b) [3 points] Sketch the impulse response $h[n]$ for this system. (Label axes.)

(c) [3 points] What is the frequency response $H(\omega)$ of this system?

(d) [3 points] How would you modify the scale factor 2 for $x[n-2]$ in this system so that the DC response $H(0) = 1$.

(e) [3 points] Assume a bounded input sequence $x[n]$ such that $|x[n]| < 1$ for all n. For such an input sequence, and for the *original unmodified* system above, what is the bound on $|y[n]|$ for the output sequence $y[n]$?

(f) [3 points] Write computer code to implement the original unmodified system. That is, write code to compute $y[n]$ for $n = 0, 1, 2, ..., N-1$, given input $x[n]$ for $n = 0, 1, 2, ..., N-1$.