GPGN 404
1st Midterm Exam
October 04, 2013 **Name:** _____

Question:	1	2	3	4	5	Total
Points:	6	6	6	16	16	50
Score:						

Question 1 ..... (6 points)

(a) What is the period N (in samples) of the sequence $x[n] = \cos(3\pi n/2)$?

(b) For the sequence $x[n] = \sin(2\pi f n)$, list all frequencies f between 0 and 1 (not including 0 and 1) for which the period is $N = 5$ samples.

Question 2 ..... (6 points)

Prove the convolution theorem. That is, prove that, if $y[n] = h[n] * x[n]$, then $Y(\omega) = H(\omega)X(\omega)$. Here, the symbol $*$ denotes convolution, and $H(\omega)$, $X(\omega)$, and $Y(\omega)$ are discrete-time Fourier transforms of the sequences $h[n]$, $x[n]$, and $y[n]$, respectively.

Question 3 ..... (6 points)

Consider the following systems $y[n] = T\{x[n]\}$. For each system, specify whether or not it is linear (L) and/or time-invariant (TI), and describe in words what the system does.

(a) $y[n] = x[3n]$

(b) $y[n] = x[-n]$

Question 4 ..... (16 points)

Consider a causal system defined by the difference equation $y[n] - \frac{3}{4}y[n-1] = \frac{1}{4}x[n]$.

(a) Complete the following computer program that implements this system

```
int nt = x.length; // lengths of arrays x and y are equal
y[0] =
for (int n=                              ) {

}
```

(b) What values $x[n]$ have you assumed for $n < 0$?

(c) Sketch the impulse response $h[n]$ for this system.

(d) Why use the difference equation above instead of simply convolving with the impulse response $h[n]$?

(e) Show that this system is stable, by finding the bound B_y on the output $y[n]$, in terms of the bound B_x on the input $x[n]$.

(f) What is the frequency response $H(\omega)$ of this system?

(g) What is the DC (zero-frequency) response $H(0)$ of this system?

(h) Write the difference equation for a similar system, with different coefficients, that is unstable.

Question 5 ..... (16 points)

Consider two linear time-invariant systems with impulse responses

$$h_1[n] = u[n] - u[n-3]$$
$$h_2[n] = h_1[-n]$$

where $u[n]$ denotes the unit-step sequence.

(a) Sketch the two impulse responses $h_1[n]$ and $h_2[n]$.

(b) For an input sequence $x[n] = \delta[n] - \delta[n-1]$, sketch outputs $y_1[n]$ and $y_2[n]$ for these two systems.

(c) Sketch the impulse response $h[n] = h_1[n] * h_2[n]$ for a system composed of the two systems describe above, where $*$ denotes convolution.

(d) For the input $x[n] = \delta[n] - \delta[n-1]$, sketch the output $y[n]$ of the composite system.

(e) How would you modify the impulse response $h_1[n]$ (here, do not overlook the subscript 1) so that the composite system with impulse response $h[n]$ has DC (zero-frequency) response $H(0) = 1$?