

**GPGN 404**  
**1st Midterm Exam**  
**February 24, 2005**

**Name:** \_\_\_\_\_

Question:	1	2	3	Total
Points:	19	11	20	50
Score:				

Question 1 ..... (19 points)

Let  $h[n] = u[n]$  (the unit-step function) be the impulse response of an LTI system.

- (a) [1 point] Is this system causal?
  
- (b) [1 point] Is this system stable?
  
- (c) [2 points] What is the  $Z$ -transform  $H(z)$  of this system?
  
- (d) [2 points] What is the region-of-convergence (ROC) for  $H(z)$ ?
  
- (e) [1 point] Where in the complex  $Z$ -plane is the (one) pole?
  
- (f) [1 point] Where in the complex  $Z$ -plane is the (one) zero?
  
- (g) [1 point] Write the linear constant-coefficient difference equation corresponding to this system, for an input sequence  $x[n]$  and output sequence  $y[n]$ .
  
- (h) [2 points] Consider the frequency response  $H(e^{j\omega})$  of this LTI system. Assuming an input sequence  $x[n] = \cos(\omega n)$ , for what frequency  $\omega$  is the output of this system unbounded?

(i) [4 points] Given the input  $x[n] = u[n]$ , what is the output  $y[n] = h[n] * x[n]$  of this system?

(j) [4 points] What is the  $Z$ -transform  $Y(z)$  of the output sequence  $y[n]$ ? (Include the ROC.)

Question 2.....(11 points)

Let  $h[n] = \frac{1}{3}(u[n + 1] - u[n - 2])$  be the impulse response of an LTI system.

(a) [1 point] Sketch this impulse response  $h[n]$ .

(b) [1 point] Is this system causal?

(c) [1 point] Is this system stable?

(d) [2 points] What is the  $Z$ -transform  $H(z)$  of this system?

- (e) [2 points] What is the region-of-convergence (ROC) for  $H(z)$ ?
- (f) [1 point] Where in the complex  $Z$ -plane is the one pole?
- (g) [1 point] Where in the complex  $Z$ -plane is the one zero?
- (h) [2 points] Qualitatively, in words, what does this LTI system (filter) do to any input sequence  $x[n]$ ?

Question 3.....(20 points)

Consider the *anti-causal* LTI system with  $Z$ -transform

$$H_1(z) = \frac{-1}{1 - 2z^{-1}}$$

and the *causal* LTI system with  $Z$ -transform

$$H_2(z) = -1 + 2z^{-1}.$$

- (a) [1 point] Where in the complex  $Z$ -plane is the one pole of  $H_1(z)$ ?
- (b) [1 point] Where in the complex  $Z$ -plane is the one zero of  $H_1(z)$ ?
- (c) [2 points] What is the region-of-convergence (ROC) for  $H_1(z)$ ?
- (d) [1 point] Is the anti-causal system defined by  $H_1(z)$  stable?

- (e) [4 points] Sketch the impulse response  $h_1[n]$ .
- (f) [1 point] Where in the complex  $Z$ -plane is the one pole of  $H_2(z)$ ?
- (g) [1 point] Where in the complex  $Z$ -plane is the one zero of  $H_2(z)$ ?
- (h) [2 points] What is the region-of-convergence (ROC) for  $H_2(z)$ ?
- (i) [2 points] Sketch the impulse response  $h_2[n]$ .
- (j) [2 points] Sketch the impulse response  $h[n] = h_1[n] * h_2[n]$  of the composite system obtained by convolving  $h_1[n]$  with  $h_2[n]$ .

(k) [2 points] What is the  $Z$ -transform  $H(z)$  of the composite system with impulse response  $h[n] = h_1[n] * h_2[n]$ ?

(l) [1 point] What is the region-of-convergence (ROC) for  $H(z)$ ?