

GPGN 404
2nd Midterm Exam
November 10, 2006

Name: _____

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|-----------|---|---|----|----|----|-------|
| Question: | 1 | 2 | 3 | 4 | 5 | Total |
| Points: | 2 | 3 | 18 | 14 | 13 | 50 |
| Score: | | | | | | |

Question 1 (2 points)
Sketch the impulse response of any digital filter with an exactly linear non-zero phase response.

Question 2 (3 points)
Sketch the impulse response of any digital filter that has an impulse response with exactly three non-zero samples and a zero-phase response.

Question 3 (18 points)

Consider the LTI digital filter defined by $H(z) = \frac{1}{2} + \frac{1}{2}z^{-2}$.

- (a) [2 points] Sketch the impulse response $h[n]$ of this filter. (Label axes.)

- (b) [3 points] Sketch the locations of the poles and zeros of $H(z)$ in the complex z -plane.

- (c) [2 points] What is the region of convergence (ROC) for $H(z)$?

- (d) [2 points] Is this system causal? Stable? Why or why not?

- (e) [2 points] What is the frequency response $H(\omega)$ of this filter?

- (f) [3 points] Sketch the amplitude response $A(\omega)$ for $-\pi \leq \omega \leq \pi$. (Label axes.)

- (g) [2 points] Sketch the phase response $\phi(\omega)$ for $-\pi \leq \omega \leq \pi$. (Label axes.)

- (h) [2 points] When applied to a digital signal with sampling interval $T = 0.5$ s, what frequency (in Hz) is most attenuated by this filter?

Question 4 (14 points)

Consider the causal LTI digital filter defined by $H(z) = (1 + z^{-2})/(1 + 0.81z^{-2})$.

- (a) [3 points] Sketch the locations of the poles and zeros of $H(z)$ in the complex z -plane.

- (b) [2 points] What is the region of convergence for $H(z)$?

- (c) [2 points] Is this system stable? Why or why not?

- (d) [3 points] Sketch the amplitude response $A(\omega)$ for $-\pi \leq \omega \leq \pi$. (Label axes.)

- (e) [2 points] Sketch the phase response $\phi(\omega)$ for $-\pi \leq \omega \leq \pi$. (Label axes.)

- (f) [2 points] To implement this filter, what difference equation would you solve?

Question 5 (13 points)

Assume that $x[n] = x_c(nT)$ is a *non-aliased* sequence obtained by uniformly sampling a continuous bandlimited signal $x_c(t)$ with sampling interval $T = 0.01$ s. (The time of first sample is zero.) By “bandlimited”, we mean that the Fourier transform of $x_c(t)$ is zero for frequencies greater than some maximum frequency F_m , in Hz.

(a) [2 points] For the specified sampling interval T , what is the Nyquist frequency (in Hz)?

(b) [2 points] What is an upper bound for the maximum frequency F_m (in Hz)?

(c) [2 points] If $y_c(t) \equiv x_c(2t)$, how would you compute the corresponding sequence $y[n] = y_c(nT)$ from $x[n]$? (Express your answer *without* a sinc function.)

(d) [2 points] For the specified sampling interval T , what maximum frequency F_m for $x_c(t)$ will ensure that $y[n]$ is not aliased?

(e) [3 points] If $z_c(t) \equiv x_c(t - T/3)$, how would you compute the corresponding sequence $z[n] = z_c(nT)$ from $x[n]$? (Express your answer *with* a sinc function.)

(f) [2 points] For the specified sampling interval T , what maximum frequency F_m for $x_c(t)$ will ensure that $z[n]$ is not aliased?