Name:

| Question: | 1 | 2 | Total |
| :--- | :---: | :---: | :---: |
| Points: | 25 | 25 | 50 |
| Score: |  |  |  |

Question 1
A digital signal $x[n]$ with a sampling interval $T=2 \mathrm{~ms}$ is contaminated with noise at two frequencies: $0(\mathrm{DC})$ and 250 Hz . Your task is to design and implement a filter to attenuate the noise, without significantly altering the amplitude and phase for other frequencies in the output sequence $y[n]$.
(a) [4 points] Determine the Nyquist frequency $F_{N}$ (in Hz ) for the input and output sequences $x[n]$ and $y[n]$.
(b) [4 points] Sketch the locations in the complex $z$-plane of two poles and two zeros for a causal and stable filter that will eliminate the noise at the two frequencies 0 and 250 Hz , while mostly preserving the signal for other frequencies.
(c) [4 points] Specify the system response $H(z)$ for your filter, including the region of convergence (ROC). Include in your $H(z)$ a scale factor so that the the amplitude response at 125 Hz is exactly one.
(d) [5 points] Sketch the amplitude and phase responses $A(F)$ and $\phi(F)$ of your filter for frequencies $-F_{N} \leq F \leq F_{N}$. Label all axes.
(e) [4 points] Write a linear constant-coefficient difference equation for your filter in terms of the input and output sequences $x[n]$ and $y[n]$. Specify numerical values for the coefficients in your equation.
(f) [4 points] Write a computer program fragment that implements your filter. Include in your fragment proper handling of the beginnings and ends of the input and output sequences $x[n]$ and $y[n]$.

The Nyquist frequency for sampled music on a CD is approximately 22 kHz .
(a) [3 points] What is the CD sampling frequency, in kHz ?
(b) [3 points] The Nyquist frequency 22 kHz was chosen because:

- typical humans cannot hear anything above 22 kHz ?
- frequencies of acoustic waves in air are less than 22 kHz ?
- both of the above?
(c) [3 points] Depending on your answer to the previous question, an analog low-pass filter may (or may not) be required before the musical recording is sampled. Why is (or is not) such a filter necessary?
(d) [4 points] Sketch the amplitude spectrum (with labeled axes) of sampled CD music, for frequencies between -44 and 44 kHz .
(e) [4 points] In his misspent youth Professor Hale listened too often to loud music, so he can no longer hear anything above 11 kHz . Describe a digital system that will convert without aliasing input sequences $x[n]$ in his music collection into output sequences $y[n]$ requiring only half the samples.
(f) [4 points] Audiophile Sam wants his music more finely sampled than on standard CDs. Write a computer program fragment that will resample his CDs so that output sequences $y[n]$ with length $n t y$ have half the sampling interval of input sequences $x[n]$ with length $n t x=n t y / 2$. (Hint: your program need not include the output/input sampling intervals, just their ratio, which is $1 / 2$.)
(g) [4 points] Sketch the amplitude spectrum (with labeled axes) of Sam's CD music after resampling with your computer program above, for frequencies between - 44 and 44 kHz .

