GPGN 404
2nd Midterm Exam
April 21, 2005

Name: ____________________

Question:	1	2	3	Total
Points:	10	17	23	50
Score:				

Question 1 .... (10 points)

Consider the digital sequence $x[n] = cos(\pi n/2)$.

(a) [2 points] Sketch this sequence. (Label axes.)

(b) [2 points] Suppose that we obtained this sequence $x[n]$ by sampling a continuous-time signal $x_c(t)$, with sampling interval $T = 0.005$ seconds. For this sampling interval, what is the Nyquist frequency, in Hz?

(c) [3 points] Suppose that our continuous-time signal $x_c(t)$ is bandlimited to the frequency range $0 \leq |\Omega| < \Omega_N$, where Ω_N is the Nyquist frequency in radians/second. Given the sampling interval $T = 0.005$ seconds, what is the continuous-time signal $x_c(t)$?

(d) [3 points] Now suppose that our continuous-time signal $x_c(t)$ is bandlimited to the frequency range $400\pi \leq |\Omega| < 600\pi$ radians/second, and that our digital signal $x[n] = cos(\pi n/2)$ and sampling interval $T = 0.005$ seconds are the same. What is the continuous-time signal $x_c(t)$?

Question 2 .... (17 points)

Consider the filter $h[n] = \frac{1}{2}(\delta[n+1] - \delta[n-1])$.

(a) [2 points] Sketch this filter. (Label axes.)

(b) [3 points] What is the Z-transform $H(z)$ of this filter?

(c) [3 points] Where in the complex z-plane are the two poles and two zeros?

(d) [3 points] What is the frequency response $H(\omega)$ of this filter?

(e) [3 points] Sketch the amplitude response $A(\omega)$ of this filter. (Label axes.)

(f) [3 points] Sketch the phase response $\phi(\omega)$ of this filter. (Label axes.)

Question 3 .... (23 points)

Consider the causal, continuous-time smoothing filter defined by:

$$h_c(t) = \begin{cases} e^{-\Omega_0 t} & ; \quad t \geq 0 \\ 0 & ; \quad t < 0 \end{cases},$$

where $\Omega_0 = \pi/2$ radians/second, and t denotes time in seconds.

(a) [3 points] What is the frequency response (continuous-time Fourier transform) $H_c(\Omega)$ of the continuous-time filter $h_c(t)$?

(b) [3 points] What is the amplitude response $A_c(\Omega) = |H_c(\Omega)|$?

(c) [3 points] Sketch the amplitude response $A_c(\Omega)$, for frequencies $-\pi \leq \Omega \leq \pi$. In your sketch, label the amplitudes at frequencies $\Omega = 0$ and $\Omega = \Omega_0 = \pi/2$ radians/second.

(d) [2 points] Create a digital filter $h[n]$ by sampling the continuous-time filter $h_c(t)$, with sampling interval $T = 1$ second. What is $h[n]$?

(e) [3 points] What is the frequency response (discrete-time Fourier transform) $H(\omega)$ of the digital filter?

(f) [3 points] What is the amplitude response $A(\omega) = |H(\omega)|$ of the digital filter?

(g) [3 points] Sketch the amplitude response $A(\omega)$, for frequencies $-\pi \leq \omega \leq \pi$. In your sketch, label the amplitudes at frequencies $\omega = 0$ and $\omega = \pi/2$ radians/sample.

(h) [3 points] For those who have not studied digital signal processing, sampling can yield surprises. In the example above, $h[n] = h_c(t = nT)$, exactly. Yet, the frequency responses $H(\omega)$ and $H_c(\Omega = \omega/T)$ are not equal, for any frequency ω. [Compare, for example, $A(\omega = 0)$ with $A_c(\Omega = 0)$ in your sketches above.] Explain.