Using Monte Carlo simulation with DCF and real options risk pricing techniques to analyse a mine financing proposal

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Abstract: This paper uses Monte Carlo simulation with both DCF and real options risk pricing techniques to evaluate an actual project financing proposal for a small gold mine. Project free cash flows accrue to equity, the host government through a royalty and corporate income tax, and creditors through a non-recourse project loan. Net present values of the contingent payouts to each project participant are calculated using real option and discounted cash flow valuation methods. The loan covenants are analysed to demonstrate how they impact value and influence the possibility of default. In particular, a gold hedging covenant is studied whereby gold put options are purchased to provide protection from low gold price environments. The put options are paid for by selling exposure to higher gold prices through call options. The analysis shows that simulation can assist with determining whether financing terms are appropriate and whether these terms can be altered to better suit the project risk profile.

Keywords: real options; project finance; gold hedge; mining; project valuation; Monte Carlo simulation; financial engineering; risk management.

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1 Introduction

Mining companies must often go to debt markets for financing. Since many debt instruments are not traded in the market, and since mining companies are not specialists in financial engineering, it is difficult for them to evaluate the terms being offered by the lender. This paper evaluates an actual financing proposal offered to a small African gold mine. The mine used traditional static (single value) and Monte Carlo discounted cash flow methods to evaluate the proposal. We then compare this with a real options view of the proposal which relies on market-based signals of price risk. A hedge programme mandated in the lending proposal is also analysed to determine how it affects the equity and government cash flows and whether its structure can be modified to increase exposure to higher gold prices while still maintaining a particular level of protection against lower gold prices. The value impacts of the loan programme are strikingly dependent on the model of the price of risk and on the method of forecasting gold prices. The first approach shows the deal as having positive Net Present Value (NPV) for the mine, while the latter approach shows the deal as transferring wealth from the mine to the lender, even on a risk-adjusted basis.

Mine valuation professionals have been actively investigating how to improve Discounted Cash Flow (DCF) valuation methods so that they can better represent mining industry complexities. Some characteristics of the mining industry that are problematic for DCF valuation include valuing long-term base metal and industrial mineral project cash flows, recognising the value and risk impact of managerial flexibility and contingent project payouts such as sliding scale royalties, and explaining the value premium placed on gold projects by the financial markets. Valuation professionals are experimenting with numerical techniques such as Monte Carlo simulation and decision tree analysis and are considering modern finance methods such as real options (RO) to more fully represent the mining business environment in a cash flow model. Salahor (1998) and Samis et al. (2006) provide detailed discussion about the differences between the DCF and RO valuation methods applied to mining and oil projects.

These efforts to improve valuation are also applicable in valuing non-equity cash flow streams such as those accruing to the government and project creditors. These cash flow streams are contingent cash flows, as their level is contingent on the profitability of the project at each point in time. Just as RO has been used to estimate and value the contingent cash flows created by managerial flexibility and sliding-scale royalties, it can also be used to estimate and value the contingent cash flows associated with taxes and project finance. Early uses of RO to value debt focused on distributing underlying company value between equity and creditors (Merton, 1974). Later examples applied these ideas to project financing, where cash flows may be distributed between equity, senior and subordinated debt, and loan guarantors (Mason and Merton, 1985; Samis, 1995). The government taxes and royalties generated by a project have been explicitly valued within a similar framework by Brennan and Schwartz (1985), MacKie-Mason (1990), Blake and Roberts (2006), and Samis et al. (2007). Recognition of managerial flexibility and its impact on project financing is discussed by Mauer and Triantis (1994), Samis (1995) and Finnerty (2007). Guj (2011) values a compound earn-in option on a mining project, which can be viewed as a mechanism by which a small firm without access to debt finances exploration and development expenditures by offering a share of its project to the lender. A conceptual framework to integrate risk management tools into project finance is provided by Gatti et al. (2007). To our knowledge, ours is the first paper to illustrate these tools at the detailed level of a case study.[1]

2 Project overview

The KuisebSun Gold Project[2] is a development-stage mine containing a total of 6.6 million tonnes of run-of-mine (ROM) ore and 545,000 troy ounces (oz) of recoverable gold. The ore will be mined in conventional open pits at an average rate of 946,000 tonnes per year (tpy) over a mine life of seven years, starting in 2008 after a one-year construction period.

Total operating costs including transportation and refining of gold are estimated to be US $300/oz or US $170/ROM tonne. Pre-production development is expected to cost US $49.9 million and take one year. Sustaining capital averages US $0.9 million per year. Production amounts, operating costs and capital expenditures are assumed to be known with certainty. As is typical in engineering appraisals of such projects, costs are assumed to be constant in real terms.[3] Production and cost uncertainty can be incorporated into the analysis via Monte Carlo modelling. Management flexibility, such as early closure in adverse business conditions, is ignored in this case study.

There are three participants in this project: equity, government and a consortium of banks. Each participant's share of cash flows is valued using real monetary terms, as is often done in engineering studies or business development valuations. Any input data that are in nominal terms (e.g. loan principal) is converted into real monetary terms.

2.1 Economic environment and gold price model

The project financing proposal takes place in the year 2007. The two valuation models, RO and DCF, require different economic data.

The real risk-free interest rate is assumed to be 2.6% for use in the RO NPV estimates.[4] This rate is estimated as a 5% nominal risk-free rate (approximate US government bond yield) less a consumer price index inflation rate of 2.4%. Gold price movements are assumed to have low correlation with financial markets. Assets with low correlation to the financial markets require only a small risk-adjustment to compensate investors for risk. The RO model assumes a risk adjustment of 0.8% for gold price uncertainty based on the Capital Asset Pricing Model (CAPM). The risk-free interest rate accounts for the time value of money.

The DCF model uses a discount rate that is assumed to reflect the aggregate riskiness of the project, typically calculated from a project Beta and the Beta of project debt. DCF NPVs for the equity, government, and creditor cash flow streams are estimated using a 7.6% real risk-adjusted discount rate, which implies a 5% risk premium. That is, the project is deemed to have the same risk as the broad market portfolio. As is typical with DCF analyses, the same DCF risk premium is applied to each participant cash flow stream.

The DCF and RO valuation models will be performed under static and Monte Carlo cash flow modelling. For the Monte Carlo model, gold price uncertainty is described over the life of the project by a discretised geometric Brownian motion diffusion process.

$$dS = \alpha S dt + \sigma S dz$$

where S is the current price, α is the instantaneous rate of drift, σ is the price volatility, and dz is a Wiener process. The discretisation is easiest to carry out in log space,

$$\ln S_{t+\Delta t} = \ln S_t + \alpha \Delta t + \sigma \sqrt{\Delta t}\, \varepsilon_t,$$

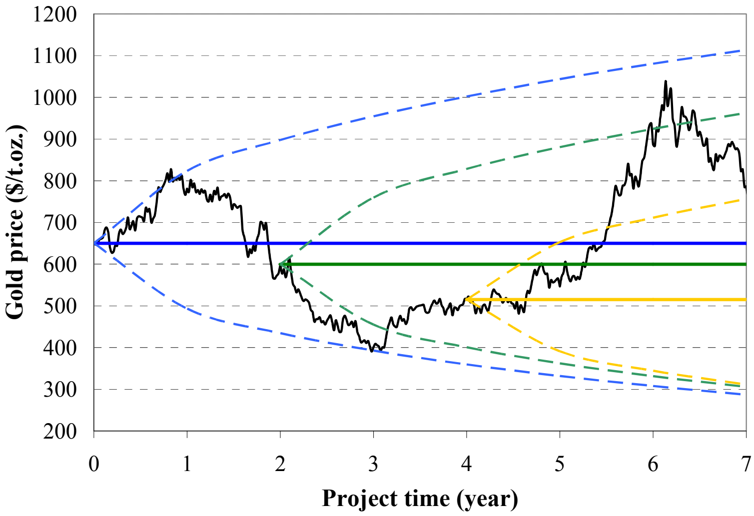
where $\varepsilon_t \sim N(0,1)$ is an independent and identically distributed random draw from the standard normal distribution and Δt is the interval of simulation as a fraction of the length of time period t (Davis, 2012). For example, if price is conceptualised as annual data, a simulation of weekly prices would set Δt = 1/52. The result is then exponentiated to obtain the simulated price. The expected log price T periods ahead is given by

$$E_0\left[\ln S(T)\right] = \ln S(0) + \left(\alpha - \frac{1}{2}\sigma^2\right)T$$

which shows the realised drift in the process is $\left(\alpha - \frac{1}{2}\sigma^2\right)$ percent per period. Hull (2005) provides a discussion of this price process for valuing financial options. An extended overview of stochastic commodity price models can be found in Geman (2005). The gold price model initially incorporates a zero drift $\left(\alpha = -\frac{1}{2}\sigma^2\right)$ real gold spot price forecast of \$652/oz. This is not based on any analysis of market data, but rather is the market expectation used by the project equity participant, since gold mining companies almost uniformly have flat real expectations for gold prices.

An important feature of this price model is the recognition that market participants update their price forecasts as new pricing information arrives. Figure 1 outlines with a jagged solid black line, a simulated price path, generated by the gold price model over the life of the project. This is just one path among many generated during a Monte Carlo simulation. The flat solid coloured lines are the conditional price expectations and the dashed lines are the 10/90% confidence boundaries associated with the simulated price path for Year 0 (blue), Year 2 (green), and Year 4 (yellow). The initial expectation is \$652/oz, but this expectation changes with spot price movements because the model is a Markov process that allows for information updating. Such updating is important in a model where the operator dynamically optimises the project as market conditions change through time. Early project abandonment is one of the main re-optimisations used (Moel and Tufano, 2002). In this case study, we ignore such updating so as not to unnecessarily complicate the analysis.

Figure 1 Updating real gold price expectations and confidence boundaries for one simulated price path using a stochastic model of gold price movements (see online version for colours)



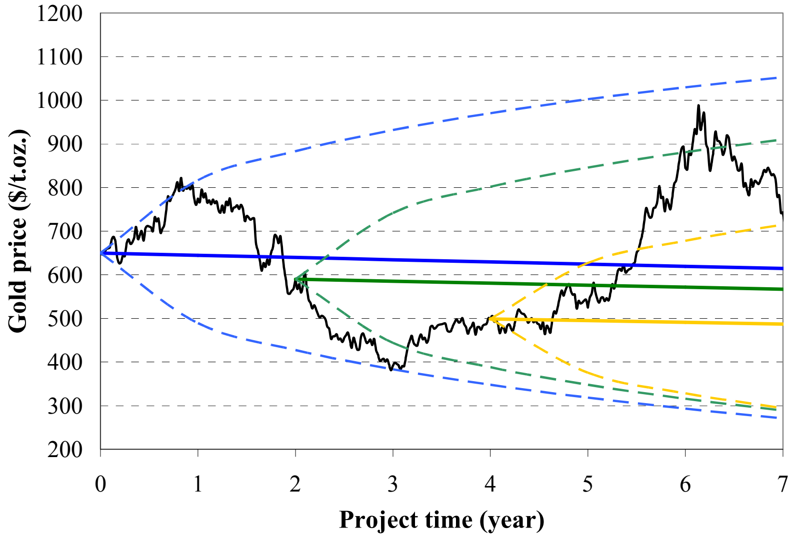
The gold price model has one other important characteristic that is relevant. The magnitude of absolute gold price uncertainty grows over time. This increase in uncertainty is illustrated by the dashed lines, which move further apart the further into the future the conditional forecast is made. This behaviour of uncertainty reflects that a constant discount gold rate risk adjustment is appropriate. We assume that it is 0.8% per year.[5] In the RO model the simulated gold prices are subject to the risk-adjustment. Figure 2 presents the risk-adjusted gold price model. The price expectations now trend downward because of the risk-adjustment, and are sometimes called risk-neutral or risk-adjusted expectations. Note that the structure in Figure 2 is a result of the gold price forecast made by the mining company and is possibly inconsistent with the contango structure displayed by the nominal gold forward curve in financial markets on the valuation date (see Figure 5 for an illustration of the nominal gold forward contango). This highlights the first level of inconsistency between traditional project valuations and modern financial techniques: the real gold price forecasts used in project valuation are often not reconciled with information from the metal forward markets.

2.2 Taxation and project financing

The host government participates in the project through a royalty and a corporate income tax. The royalty is 3% of gross revenues less gold refining costs. A 25% corporate income tax is levied on net operating profits, calculated as revenues less operating costs, interest and royalty payments, depreciation, and tax loss carry forwards. It is assumed that tax losses can be carried forward indefinitely to simplify the calculation.

A bank consortium has agreed to provide a Project Development Loan (PDL) of $35.0 million that is to be repaid over the first four years of production. Equity provides the remaining $14.9 million of development capital. The loan carries an annual nominal interest rate of 8% that is amortised over four years with nominal annual year-end payments of $10.6 million. The principal and interest components of these payments are converted into real monetary terms using a 2.4% inflation rate calculated from the US Consumer Price Index (CPI). Scheduled interest payments total $6.9 million in real terms over the life of the loan.

Figure 2 Updating real gold price expectations and confidence boundaries for one simulated price path using a stochastic model of risk-adjusted (risk-neutral) gold price movements (see online version for colours)



The loan is non-recourse so that the consortium can only claim interest and principal payments from the project cash flows. The project is in technical default of the loan provisions when it is unable to pay scheduled interest and principal payments. The consortium does not foreclose on the project when a technical default event occurs. Instead, unpaid interest and principal is transferred to a Loan Shortfall Facility (LSF) to be carried forward at a nominal compounded 11% interest rate until there is enough residual project cash flow for repayment. In a given year, a positive LSF balance will be paid down by any positive residual project cash flow after the payment of operating and refining costs, royalties, corporate income tax, scheduled PDL interest and principal payments, and sustaining capital. Any positive LSF balance at the end of the project represents a shortfall in contracted payments to the bank consortium and is considered a full loan default event. The LSF is a notional creditor asset because it records the outstanding balance of unpaid interest and principal owed to the creditors. Project cash flows used to pay down the outstanding balance are actually paid to the creditors. Note that LSF cash flows may occur after the fourth year if there is an outstanding balance at this time.

The bank consortium also requires the project owners to enter into a hedging agreement where a series of European gold call options are sold and the proceeds are used to purchase gold put options on 100% of gold production. This agreement provides protection against the possibility of gold price declines and guarantees that the project will generate sufficient funds to repay the project loan.

The banking consortium's risk manager offers a zero-value put-call collar to the project operator. The risk manager suggests that call options must be sold by the project owners on 54% of the gold produced in each of the first four years of mine operation at a strike price of $700/oz. These options give the purchaser the right but not the obligation to buy gold for $700/oz at times and in quantities reflecting the projects expected production profile. The fees from selling gold call options are then used by the project owners to purchase European put options with a strike price of $630/oz on 100% of the gold produced in the first four years of production. These put options provide the project owners with the right but not the obligation to sell up to 100% of the gold produced to a third party for $630/oz at the end of each year.

The gold call and put option exercise prices are nominal figures and are converted into real monetary terms using the 2.4% inflation rate. For example, the real call and put exercise prices are $635.92/oz and $572.33/oz, respectively, in the fourth year of the project.

3 Valuation and risk assessment results

This section provides estimates of static and Monte Carlo DCF and RO NPVs of each project stakeholder cash flow stream, Internal Rates of Return (IRR) for select cash flow streams, estimations of default probabilities and their impact on value, and an investigation of the gold hedging programme's influence on value and risk. While many would argue that modern finance requires the use of RO to correctly value each participant's cash flow streams, we carry the DCF analysis throughout to show the degree of difference between the two types of analysis. While the valuation of the equity cash flows should be conducted with a DCF discount rate that reflects varying financial leverage for the project through time (Ruback, 1995; Ruback, 2002), we follow the standard approach of using the same DCF discount rate regardless of the loan position.

3.1 Net present values when there is no hedging programme in place

The DCF and RO NPV results are presented in Table 1. The static DCF results are estimated with the initial gold price forecast in Figure 1 and using a real discount rate of 7.6% for all cash flow items, even though various individual cash flow streams will have different risks (Jacoby and Laughton, 1992). The static RO results are estimated with the risk-adjusted initial price forecast in Figure 2 and a 2.6% discount rate (the real risk-free rate) for all cash flow items. The Monte Carlo simulations add explicit price simulations. The reference project NPV given in the top row of the table is the value of the project cash flows when there are no taxes, royalties, or non-equity financing. This stream is defined as project revenue less operating and refining costs, capital expenditure, and working capital. Reference project NPV is the underlying project value that is available to be distributed between project stakeholders based on the terms of taxation and finance. The sum of equity, royalty, corporate income tax, and project finance NPVs in a particular column is equal to the reference project NPV. This construct is consistent with the Modigliani–Miller theorem (1958) which hypothesises that financing distributes value but does not create it.

Table 1 Static and Monte Carlo net present values under DCF and RO discounting techniques

Cash flow stream	*DCF NPV ($ million)*		*RO NPV ($ million)*	
	Static	*Monte Carlo*	*Static*	*Monte Carlo*
Reference project	94.0	94.0	112.2	112.2
Equity	65.7	66.2	75.4	75.9
Government royalty	8.1	8.1	9.3	9.3
Corporate income tax	22.1	22.8	25.2	26.1
Project development loan	–1.9	–4.6	2.4	–1.2
Loan shortfall facility	0	1.5	0	2.1

The NPV of a particular cash flow stream varies depending on whether DCF or RO risk adjustments are used. In this case the RO NPVs are larger than the DCF NPVs, reflecting that the effective discount rate associated with the RO method is less than 7.6%. The higher RO values are consistent with the gold project value premium observed in the financial markets. This premium is the difference between the value placed on a gold project by the market and the value calculated by the DCF method using a standard risk-adjusted discount rate. Some finance industry participants account for the premium by using a 0% discount rate to value gold projects or by multiplying DCF NPVs by a qualitative market multiple.

Using the static DCF technique the PDL has a negative NPV to the lender. This results from the loan having interest payments based on a 5.4% real interest rate and the loan present value calculated by discounting cash flows at a 7.6% real discount rate. The present value of the loan will be zero at a 5.4% discount rate. Under the static RO technique, the amortised loan payments have a positive $2.4 million value to the lender. This indicates that the loan rate is too high – the correct discount rate for a loan that has no default risk, as is implicitly modelled in the static case, is the risk-free rate, in this case 2.6% real. The LSF has no value in the static cash flow models as it is never used based on the single, expected path for the price of gold as shown in Figures 1 and 2.

The reference project and government royalty NPV do not change between static and Monte Carlo models since both these streams have a linear relationship with gold price and no path dependencies. The NPVs of other cash flow streams differ under simulation because of non-linearities and path dependencies. A non-linear cash flow pay-off is one that varies non-linearly across the simulated variable. An example is corporate income tax, whereby the project pays a tax to the government when operating profit is positive but only receives the present value benefit of tax loss carry forwards when profits are negative. A path-dependent cash flow is one whose magnitude is conditional on the path followed by the simulated variable up to that point. For example, the facility to carry forward tax losses will not affect the tax cash flow calculations late in the project after a period of high gold prices since it is likely that there will be no tax-losses to carry forward. Those same tax cash flow calculations will be affected by the tax loss carry forwards after a period of low gold prices in which operating losses were made. Monte Carlo simulation is able to recognise the effect of cash flow non-linearity and path dependency on estimated cash flow levels because it samples cash flows across a range of gold prices over time. Static cash flow models, on the other hand, miss these impacts because they calculate cash flow for only a single gold price path, the expected price.

The impact of non-linearity and path dependence is best observed in the corporate income tax and LSF values in Table 1. The corporate income tax is underestimated in a static analysis. This is also true of the LSF. The Monte Carlo simulation recognises that some low gold price environments may occur during the project such that a technical default occurs and the unpaid balance of interest and principal is transferred to the LSF to be carried forward and possibly paid-off when profitability recovers. The static cash flow model does not record these value effects because it does not recognise the possibility that the LSF is used. The positive LSF value in each Monte Carlo valuation is a demonstration of this recognition. Using Monte Carlo simulation, the combined creditor cash flows have a DCF NPV of –\$3.1 million and an RO NPV of \$0.9 million. It is interesting to note that the static cash flow models overvalue the PDL from the lender's perspective. This is because there are situations where there is still default on the loan during unfavourable price simulations.

Table 2 presents the discretely compounded real IRRs for selected static and simulated cash flows streams.[6] Real IRR is the same for the project regardless of whether the DCF or RO method is used to calculate NPV because it is calculated from expected cash flow prior to the application of discounts for risk and time. The Monte Carlo analysis does not affect the reference project IRR because the two cash flow streams have the same expectations. The equity cash flow stream has a higher IRR than the Reference Project because of the leveraging effect of debt. Once again, Monte Carlo simulation picks up the fact that creditor returns are affected by the possibility of default in low gold price environments. The static model estimates the creditor cash flow IRR, which may include LSF repayments if the LSF is available, to be 5.4%, since at this discount rate the present value of the amortised loan payments equals the initial loan amount, whereas Monte Carlo estimates that the loan has either a 1.5% or 3.9% IRR depending on whether the LSF is available.

Table 2 Real discretely compounded IRR for select cash flow streams

Cash flow stream	*Internal rate of return (%)*	
	Static	*Monte Carlo*
Reference project	57.2%	57.2%
Equity without LSF	108.1%	109.3%
Creditor without LSF	5.4%	1.5%
Equity with LSF	108.1%	108.8%
Creditor with LSF	5.4%	3.9%

The lower real IRRs under the Monte Carlo analysis come from the non-linearity of loan payments, where loan payments are as expected in high gold price environments, but may not be made in full in low gold price environments. That is, the Monte Carlo analysis uncovers default risk. In this case study, the scheduled interest and principal payments are \$39.8 million in real terms. However, using the Monte Carlo simulation, the expected real cumulative creditor cash flows over the life of the project are estimated to be \$36.3 million when there is no LSF and \$38.6 million with the LSF because of the possibility of a loan default. The LSF increases the IRR of creditor cash flows because it provides an opportunity for creditors to recover unpaid interest and principal payments from future project cash flows. It also lowers the IRR of the equity participant because they are not permitted to default on the loan.

Tables 1 and 2 highlight that Monte Carlo analysis can quantitatively recognise the adverse impact of loan default on the value and return of the creditor cash flow stream. The negative effect of loan default is partially mitigated by the presence of the LSF which allows creditors to recover some or all of the missed payments from residual future cash flows. These results highlight the importance of using a valuation model that recognises the possibility of loan default in its calculation and the provisions allowing creditors to recover any cash flow shortfalls. The static cash flow model does not incorporate price scenarios in which technical default occurs and so does not recognise how default affects creditor returns. This effect is similar to the need to perform simulation to pick up the full effects of price-contingent royalties in a mining project (Samis et al., 2007).

3.2 Default probabilities

Monte Carlo simulation also supports the analysis of the likelihood of loan default. Table 3 provides estimates of the annual probability of technical default in the first four years of production. The probability of technical default rises in the third and fourth years because operations are in an area where costs are higher and gold grades are slightly lower than the first two years. The probability of at least one technical default over the four year loan term is approximately 54%.

Table 3 Annual technical loan default probabilities

Project year	*1*	*2*	*3*	*4*
Probability of technical default (%)	7.7%	3.9%	20.0%	34.5%

Technical default causes the LSF to kick in. A positive LSF balance at the end of the project indicates an actual loan default because this is the amount that the creditors are owed under the project financing terms. The probability of a positive terminal LSF balance is 15% with an expected balance of $2.1 million. Table 2 shows that the LSF increases the yield to the creditor to 3.9% compared to 1.5% without the LSF.

3.3 Hedge programme analysis using flat gold price expectations

The provider of the hedge programme can be considered a separate project participant for valuation purposes. The hedge position pays cash into or receives cash from the project based on gold price fluctuations and the aggregated call and put option positions of the programme. Its presence will affect the value of the equity, creditor, and government project participation but does not affect the reference project value since it does not change the operating project cash flows. A positive hedge programme NPV reduces the aggregate value of equity, creditor, and government cash flow streams since on a risk-and-time discounted basis the hedge programme receives more project cash flow from the call options than it pays into the project through the put options. Conversely, a negative hedge programme NPV increases the aggregate value of the other participants since it pays more cash into the project for distribution to the other participants through the put options than it receives from the call options on a time-and-risk discounted basis.

In this case study, the Monte Carlo simulation under DCF valuation shows that the hedge programme is expected to contribute a net $2.9 million to project cash flow when the flat real gold price expectations model is used (Table 4). This contribution increases average revenue per unit of gold produced from $652/oz to $657/oz. Figure 3 presents the impact on aggregate cash flows distributed to equity, creditors, and the government. The solid blue line outlines distributed cash flow when there is no hedge programme while the dashed orange line delineates distributed cash flow when the hedge programme is present. The net positive impact on project cash flows declines with time because the option exercise prices are declining in real terms.

Figure 3 Aggregate expected equity, creditor, and government cash flows, with (dashed line) and without (solid line) the hedge programme (see online version for colours)

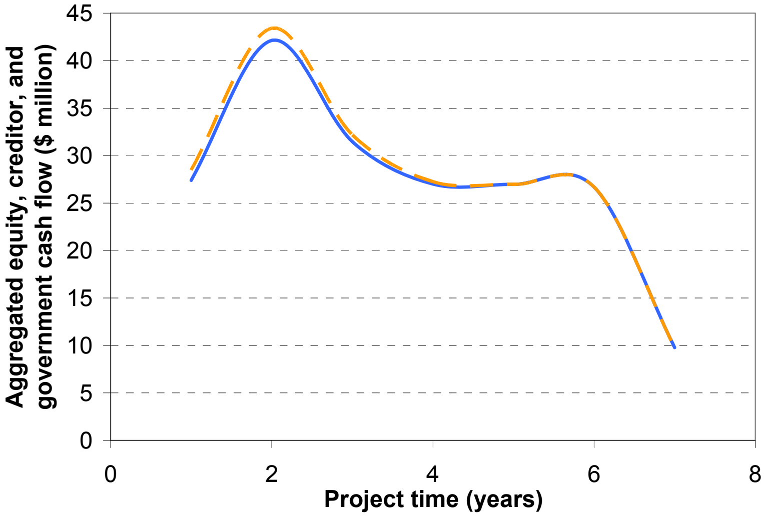


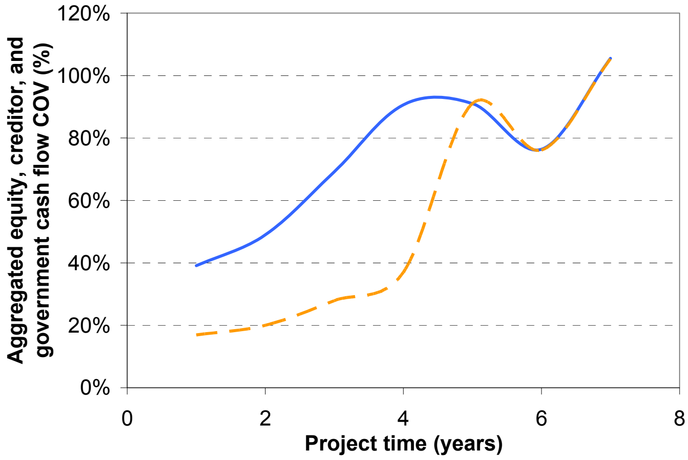
Table 4 provides the revised equity, creditor, and government DCF and RO NPVs after implementing the hedge programme. The no-hedge values as the same as the Monte Carlo values presented in Table 1. The NPV of the hedge programme's cash flow contribution is –\$2.9 million using DCF and –\$6.0 million using RO from the perspective of the programme counterparty. The DCF and RO NPVs of the equity, creditor, and government cash flow streams increase as a result of the hedge programme.

Table 4 Monte Carlo net present values with and without gold hedge programme in place under DCF and RO discounting techniques

Cash flow stream	*DCF NPV($ million)*		*RO NPV ($ million)*	
	No hedge	*With hedge*	*No hedge*	*With hedge*
Reference project	94.0	94.0	112.2	112.2
Equity	66.2	67.8	75.9	79.6
Government royalty	8.1	8.1	9.3	9.3
Corporate income tax	22.8	22.9	26.1	26.9
Project development loan	–4.6	–1.9	–1.2	2.4
Loan shortfall facility	1.5	0	2.1	0
Hedge programme	N/A	–2.9	N/A	–6.0

The hedge programme also affects the magnitude of uncertainty in the combined equity, creditor and government cash flows. Figure 4 compares the aggregated cash flow's Coefficient of Variation (CoV) with and without the hedge programme. The CoV is defined as the standard deviation of an uncertain variable divided by its expected value. It provides an indication of cash flow dispersion (the higher the CoV the greater the uncertainty) and is used here to highlight the level of uncertainty in these aggregated cash flows. The solid blue line delineates cash flow CoV when there is no hedge programme and the dashed orange line outlines cash flow CoV over project the years 1 through 5 when the hedge programme is in effect. Overall, the hedge programme reduces the average CoV for the aggregated cash flows from 74% to 54% over the project's lifetime.

Figure 4 Coefficient of variation of aggregated expected equity, creditor, and government cash flows with (dashed line) and without (solid line) the hedge programme (see online version for colours)



The hedge programme has the intended effect of reducing the probability of loan default to zero given the model assumptions and project structure because the put options provide enough additional cash when the gold price drops below $630/oz to ensure that loan payments are made in full. The value of the LSF falls to zero as a result. Creditor IRR naturally rises to 5.4%, the interest rate on the loan (see Table 5), and equity IRR increases to 113.9% due to the net wealth transfer from the hedge programme to the project.

Table 5 Real discretely compounded IRR for select cash flow streams with and without gold hedge facility in place

Cash flow stream	*Internal rate of return (%)*	
	No hedge	*With hedge*
Equity without LSF	109.3%	113.9%
Equity with LSF	108.8%	113.9%
Creditor without LSF	1.5%	5.4%
Creditor with LSF	3.9%	5.4%

If the lending bank itself offers the hedge programme, the value of the combined instruments goes from –$3.1 million to –$4.8 million in the DCF analysis and from $0.9 million to –$3.6 million in the RO analysis (Table 4). One might immediately suspect a hedge programme that has a negative NPV for the party that is offering it. We will revisit this issue in the next section.

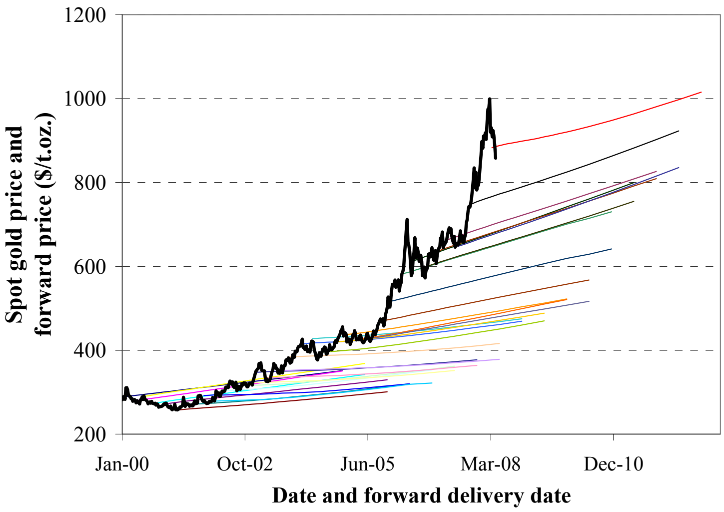
3.4 Hedge programme analysis using the gold forward curve

An obvious question is why a counterparty or the lending bank itself would enter into a hedge programme that has negative DCF and RO NPVs (Table 4). A partial answer is found in the difference between the gold price model assumptions often used in engineering or business development valuations and those used in financial markets to value financial portfolios of gold options and other derivative financial instruments. Engineers and business analysts often use a flat real gold price forecast to value future gold-linked cash flows. Financial market specialists, however, use the gold forward curve to value cash flows arising from gold derivatives.

The gold forward curve outlines a set of prices at which market participants are willing to buy or sell gold over a range of future transaction dates. These prices are called forward prices and they may be interpreted as risk-adjusted expected prices. The gold forward curve provides a market estimate of expected prices in the future given adjustments for risk and market characteristics such as the gold lease rate (Geman, 2005). This curve and its implied expectations change with spot gold price movements, reflecting a revision in investor expectations after receiving new information.

What is important for valuation purposes is the structure of the forward curve. The gold forward curve rises in nominal terms (i.e. incorporates inflation) with increasing time to the future delivery of gold. Figure 5 presents the spot price (thick black line) and gold forward curves (thin coloured lines) from 1 January 2000 to 31 March 2008 on a quarterly basis. In all cases, the forward curve rises with term.

Figure 5 Spot gold price and gold forward curves at the start of each quarter from 1 January 2000 (see online version for colours)



Source: Bloomberg Data service

The flat real gold price expectations model can be adjusted to be consistent with increasing nominal gold forward prices in the financial markets. This adjustment is derived from an arbitrage relationship which requires the gold forward price to increase with term at a rate equal to the nominal risk-free rate less a convenience yield or cost-of-carry (Hull, 2005; Geman, 2005). The convenience yield is assumed to equal the gold lease rate of 0.4% in this paper. The resulting nominal forward curve is converted into real terms by discounting the curve at the CPI inflation rate. A market-based estimate of expected gold prices can then be derived by adjusting the real gold forward curve with a gold CAPM risk-adjustment. The forecast can now be used in the DCF analysis, which maintains the same discount rate as in the previous sections.

Without the hedge programme in effect, under this model of gold prices the expected project revenue per unit of gold produced rises to $736/oz. Reference project DCF and RO NPVs increase to $122.3 million and $146.2 million, respectively (Table 6). The probability of at least one technical default on interest and principal payments decreases from 53% to 40%. The expected terminal LSF balance is $1.1 million and the probability of a positive balance (i.e. LSF default) is 8%. The loan facility provides $1.7 million of value to the lender under the RO model.

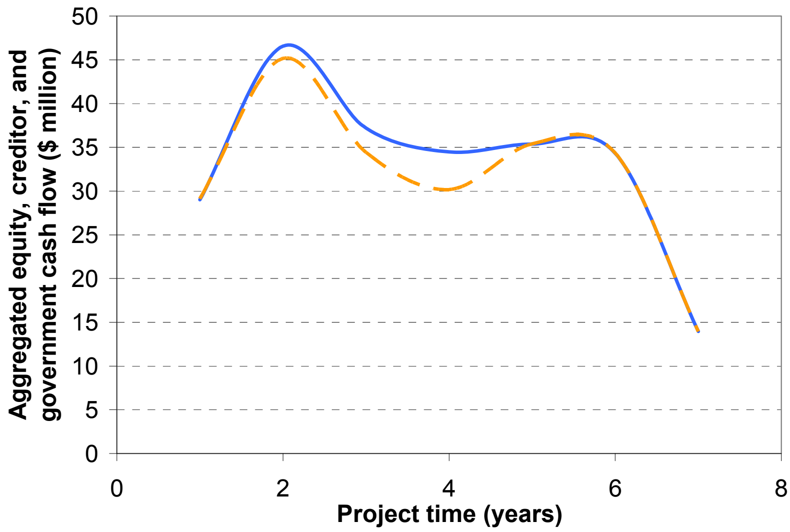
Table 6 Monte Carlo net present values given a gold forward curve price model and DCF and RO discounting techniques

Cash flow stream	*DCF NPV($ million)*		*RO NPV ($ million)*	
	No hedge	*With hedge*	*No hedge*	*With hedge*
Reference project	122.3	122.3	146.2	146.2
Equity	86.6	81.5	100.3	96.5
Government royalty	8.9	8.9	10.3	10.3
Corporate income tax	29.3	27.4	33.9	32.3
Project development loan	–3.7	–1.9	0.0	2.4
Loan shortfall facility	1.2	0	1.7	0
Hedge programme	N/A	6.4	N/A	4.7

Given the probability of loan default the lender could still argue for a hedge programme. With the gold forward curve price model and the same hedge terms as in the analysis in the previous section, the hedge programme counterparty is now expected to be the recipient of a net $6.4 million from the project cash flows in a DCF analysis and $4.7 million in an RO analysis. This hedge programme status change, from being a net contributor to project cash flow under the flat gold price model to being an expected net receiver of project cash flow under this model is due to the change in the price model parameters; the gold forward curve price model rises in real terms against put and call exercise prices that are declining in real terms. As a result, the call options held by the hedge programme counterparty are more likely to be exercised and the put options less likely to be exercised, which now produces an expected net cash outflow from the project. The positive hedge programme NPV results in a decrease in the DCF (RO) NPV of equity and government cash flow of $5.1 ($3.8) million and $1.9 ($1.6) million, respectively. The hedge programme reduces expected project revenue per unit of gold produced to $724/oz when it is implemented. The combined value of the PDL and LSF increases by $0.6 ($0.7) million after the hedge programme is implemented. Hence, the lending bank is in favour of the hedge position under this model of gold prices. The fact that the hedge programme adds value to a bank's position on the loan is good reason why the bank would want to put the hedge programme in place. In this case the bank itself offered this 'costless' collar. This suggests that project owners may want to have financing and metal hedging services provided by different investment banks in order to make transparent the costs of project risk management.

The net impact on the aggregate cash flows for distribution to equity, creditors, and the government is displayed in Figure 6. The solid blue line delineates distributed cash flow when there is no hedge programme and the dashed orange line represents distributed cash flow when the hedge programme is present. The negative impact on project cash flows increases with time because gold price expectations are increasing against declining option exercise prices in real terms.

Figure 6 Aggregated expected equity, creditor, and government cash flows with (dashed line) and without (solid line) the hedge programme in place and using the gold forward curve price model (see online version for colours)



The original hedge programme provided put protection for 100% of gold production (374,500 oz) while selling call options on 54% of gold production (202,230 oz). However, simulation is a tool which can help assess whether put protection is actually required for 100% of gold production as a means of preventing loan default. This is a reasonable question to ask give that the current put programme ensured zero loan default. It is also a question that affects equity value, since each purchase of a put option reduces equity's exposure to the possibility of higher gold price movements because this purchase is financed through the sale of call options. Project managers may decide that they would prefer to maximise their exposure to higher gold price situations while still maintaining protection against loan default. They can do this by reducing put coverage in each year and testing through simulation whether the lower level of put protection provides sufficient default protection.

Table 7 presents an amount of put protection required in each year to reduce annual loan default probabilities to below 1% given the model assumptions and the original put and call exercise prices. Overall, put options need only be sold on 55% of gold production (205,200 oz) to protect against loan default. This revised level of protection requires that call options only be sold on 30% of gold production (112,350 oz) to finance this protection. This hedge product has a zero RO value under this model, and thereby increases the equity holder's position in the project to $100 million and increases the government position from corporate income tax to $33.5 million. The net gain is the lost $4.7 million that the provider of the hedge gives up. In addition, the interest rate on the PDL, which includes default risk, should now be reduced to the risk-free rate since there is no chance of loan default. This would transfer an additional $2.4 million to the equity and government participants in the project.

Table 7 Approximate lower boundary of production coverage with put options that generates an annual loan default probability of less than 1%

Project year	*1*	*2*	*3*	*4*
Put production coverage (%)	48%	33%	60%	82%

4 Conclusions

This paper analyses a project financing proposal for a small African gold mine using both static and Monte Carlo simulation cash flow models, and using both DCF and real options techniques for discounting cash flow risk. The value of each participant's cash flows differs under the RO and DCF techniques. The differences are often substantial.

Monte Carlo simulation was used to quantify the effect of technical default risk and the ability of creditors to recover lost cash flows via a loan shortfall facility. The Monte Carlo simulation was also able to produce default probabilities and expected creditor losses. The static cash flow built around a 1-point gold price forecast could not quantify these items and analysts using this technique would need to rely on a qualitative approach to assess the value and risk effects of default.

The hedge programme mandated by the lender was also analysed with Monte Carlo simulation. Simulation showed that the number of call options sold to pay for put option protection against low gold prices could be reduced by about 50% while still maintaining the same level of default protection. This is an important consideration since gold company capitalisation is allegedly sensitive to the proportion of production that is hedged. The model also reveals that the government is a passive project participant who is negatively affected when there are project wealth transfers to the lending and hedging providers.

Monte Carlo simulation combined with real options analysis is a powerful tool for understanding how financing distributes value and risk among project stakeholders. Creditor concerns about default and equity concerns about protective loan covenants can be compared within a model to balance their competing interests. Such a comparison should promote better understanding of the trade-offs that are part of developing a plan for financing a mining project.

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Notes

1 The Excel file containing the detailed cash flow model is available from the authors upon request.

2 This case study is based on an actual mining project with a few liberties taken with the actual details. The input data for this case study were all obtained from public sources such as SEDAR, company annual reports, and conference proceedings or are assumptions based on professional judgement. The authors have not worked with any of the organisations associated with this project and have not had access to private project information while developing this case study. The results and conclusions in this paper are not a professional valuation opinion regarding the economic potential or risks of any particular mining project and cannot be relied on as such by the reader.

3 The worksheet has a switch that converts all real values to nominal dollars, if desired, using the consumer price index. The income tax worksheet is always conducted in nominal dollars. This makes the valuation results independent on whether real or nominal analysis is used.

4 All rates are continuously compounded unless otherwise noted.

5 Using a capital asset pricing model of risk, the market price of risk is estimated to be 0.40 at the time of the analysis, the correlation of gold price movements with the broad market is estimated to be 0.10, and the gold price volatility is estimated to be 0.20 based on historical returns. The risk parameter could also be recovered from gold futures options trading at the time.

6 In the Monte Carlo analyses, the IRRs are taken over the expected cash flows. Strictly speaking one should take the expected IRR over all trials (Lewellen and Long, 1972), though this is problematic since there are many trials for which the IRR is negative or non-existent. This is a general problem with the notion of IRR in stochastic environments, and leads us to prefer the NPV analyses as a tool for understanding the effects of project finance on participants in the project.