

The joint nonhyperbolic moveout inversion of PP and PS data in VTI media

Vladimir Grechka* and Ilya Tsvankin†

ABSTRACT

Nonhyperbolic moveout of P-waves in horizontally layered transversely isotropic media with a vertical symmetry axis (VTI) can be used to estimate the anellipticity coefficient η in addition to the NMO velocity $V_{\text{nmo},P}$. Those two parameters are sufficient for time processing of P-wave data (despite a certain instability in the inversion for η), but they do not constrain the vertical velocity V_{P0} and the depth scale of the model. It has been suggested in the literature that this ambiguity in the depth-domain velocity analysis for layer-cake VTI media can be resolved by combining long-spread reflection traveltimes of P-waves and mode-converted PSV-waves.

Here, we show that reflection traveltimes of horizontal PSV events help to determine the ratio of the P- and S-wave vertical velocities and the NMO velocity of SV-waves, and they give a more accurate estimate of η . However, nonhyperbolic moveout of PSV-waves turns out to be mostly controlled by wide-angle P-wave traveltimes and does not provide independent information for the inversion. As a result, even for a single-layer model and uncommonly large offsets, traveltimes of P- and PSV-waves cannot be inverted for the vertical velocity and anisotropic parameters ϵ and δ . To reconstruct the horizontally layered VTI model from surface data, it is necessary to combine long-spread traveltimes of pure P and SV reflections.

INTRODUCTION

A major complication caused by anisotropy in velocity analysis is the uncertainty in estimating the vertical velocity and depth scale of the model from surface P-wave data. For horizontally layered VTI media, P-wave reflection traveltimes are fully determined by the vertical (zero-offset) time and two interval parameters—the NMO velocity from a horizontal reflector $V_{\text{nmo},P}$ and the anellipticity coefficient η (Alkhalifah and Tsvankin, 1995). The parameters $V_{\text{nmo},P}$ and η are expressed through the vertical velocity V_{P0} and Thomsen (1986) anisotropic coefficients ϵ and δ in the following way:

$$V_{\text{nmo},P} = V_{P0} \sqrt{1 + 2\delta}; \quad (1)$$

$$\eta \equiv \frac{\epsilon - \delta}{1 + 2\delta}. \quad (2)$$

The velocity $V_{\text{nmo},P}$ can be found using hyperbolic semblance analysis of conventional-spread reflections from horizontal interfaces, while estimation of η requires inverting either long-spread (nonhyperbolic) moveout or the NMO velocities

of dipping events (Alkhalifah and Tsvankin, 1995; Grechka and Tsvankin, 1998; Tsvankin, 2001).

Therefore, even though P-wave phase and group velocities depend on V_{P0} , ϵ , and δ individually (e.g., Tsvankin, 2001), in general none of these three parameters can be obtained from P-wave reflection moveout. As a result, velocity analysis of P-wave reflection traveltimes does not provide enough information for building VTI models in depth. Although the presence of lateral heterogeneity (e.g., dipping interfaces) in the overburden makes the two-parameter description of P-wave moveout invalid, stable P-wave inversion for V_{P0} , ϵ , and δ requires substantial dip or curvature of intermediate interfaces or multiple dips within the layers (Grechka et al., 2002).

To perform depth-domain velocity analysis in VTI media, reflected PP-waves can be combined with shear-wave data. As shown by Tsvankin and Thomsen (1995), the joint inversion of long-spread (nonhyperbolic) moveout of PP- and SS-waves yields the vertical velocities V_{P0} and V_{S0} of P- and S-waves and the coefficients ϵ and δ . In multilayered VTI media, nonhyperbolic moveout of PP- and SS-waves can be inverted for the interval VTI parameters and the layer

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*Formerly Colorado School of Mines, Center for Wave Phenomena, Golden, Colorado; presently Shell International Exploration and Production Inc., Bellaire Technology Center, 3737 Bellaire Blvd., Houston, Texas 77001-0481. E-mail: vladimir.grechka@shell.com.

†Colorado School of Mines, Center for Wave Phenomena, Department of Geophysics, Golden, Colorado 80401-1887. E-mail: ilya@dix.mines.edu.
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thicknesses. (Hereafter, S denotes SV-waves; P- and SH-waves in horizontally layered VTI media are decoupled.)

Shear waves, however, are not excited in offshore surveys and are seldom used on land because of cost considerations and problems with data quality (e.g., statics). The question addressed here is whether it is possible to replace pure SS-waves in moveout inversion for horizontally layered VTI media by long-offset mode-converted PS data, as suggested by Li and Yuan (1999).

The basic difference between the long-spread moveout of pure and converted waves can be illustrated using the simplest model of a single horizontal layer (Figure 1). For PS-waves in isotropic media, the maximum shear-wave reflection angle φ_S that corresponds to infinitely large offsets is defined by Snell's law:

$$\max \varphi_S = \sin^{-1} \left(\frac{V_S}{V_P} \right), \quad (3)$$

where V_P and V_S are the velocities of P- and S-waves, respectively. If the velocity ratio V_S/V_P takes a typical value of 0.5, the angle φ_S reaches only 30° , and it is even smaller for loosely consolidated sediments with low shear-wave velocities.

This estimate of the maximum φ_S remains qualitatively valid in the presence of moderate anisotropy. For example, in a typical VTI model with the parameters $V_{P0}/V_{S0} = 2.5$, $\epsilon = 0.20$, and $\delta = 0.05$, the group angle φ_S for the offset-to-depth ratio $X/D = 3$ is equal to 38° , while the corresponding phase angle is just $\theta_S = 16^\circ$. Such angles φ_S and θ_S are insufficient to create measurable deviations of the S-wave portion of converted-wave moveout from a hyperbola (Tsvankin and Thomsen, 1994), even for uncommonly large offsets.

Since the success of the Tsvankin-Thomsen (1995) inversion algorithm was ensured by including wide-angle (up to and beyond 45°) SS-wave reflections, we can expect the replacement of SS data with PS-waves to be inadequate. In fact, the above estimates of the reflection angles suggest that long-spread moveout of PS-waves is largely governed by the wide-angle traveltimes of P-waves (which can be recorded independently) and may not provide additional information for the inversion. Below, we substantiate this conclusion by numerical testing and quantify the uncertainty in the inverted VTI parameters.

INVERSION OF PP MOVEOUT AND CONVENTIONAL-SPREAD PS DATA

As discussed above, the inversion of nonhyperbolic (long-spread) PP-wave moveout in a horizontal VTI layer yields

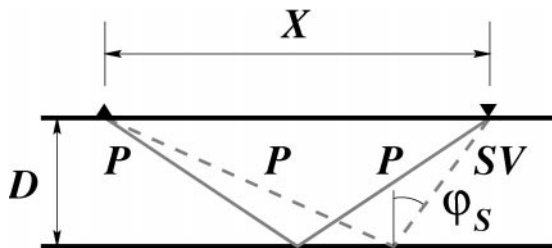


FIG. 1. PP (solid) and PS (dashed) reflections from a horizontal interface. The reflection group angle of the converted PS-wave is denoted by φ_S .

the P-wave NMO velocity $V_{\text{nmo},P}$ and the coefficient η . The trade-off between these parameters, however, may lead to sizable errors in η which can reach ± 0.1 , even for this simple model (Grechka and Tsvankin, 1998). Numerical and field examples of nonhyperbolic moveout analysis of PP-wave data are given by Alkhalifah (1997), Le Bougeant et al. (1997), Grechka and Tsvankin (1998), Toldi et al. (1999), Tsvankin (2001), and others.

First, suppose that long-spread PP reflections are supplemented with conventional-spread (i.e., the maximum offset is close to the reflector depth) traveltimes of PS-waves. Then hyperbolic velocity analysis of the PS data can provide the vertical PS traveltimes $t_{PS0} = (t_{P0} + t_{S0})/2$ (t_{P0} and t_{S0} are the two-way vertical times of the PP- and SS-waves) and the PS-wave NMO velocity. Using the Dix-type formula for the NMO velocities of pure and converted modes (Seriff and Sriram, 1991),

$$2t_{PS0} V_{\text{nmo},PS}^2 = t_{P0} V_{\text{nmo},P}^2 + t_{S0} V_{\text{nmo},S}^2, \quad (4)$$

and taking into account that $t_{S0} = 2t_{PS0} - t_{P0}$, we can obtain the NMO velocity $V_{\text{nmo},S}$ of SS (SVSV)-waves given by

$$V_{\text{nmo},S} = V_{S0} \sqrt{1 + 2\sigma}, \quad (5)$$

$$\sigma \equiv \left(\frac{V_{P0}}{V_{S0}} \right)^2 (\epsilon - \delta).$$

Also, the vertical-velocity ratio can be estimated from the vertical traveltimes t_{P0} and t_{S0} :

$$\frac{V_{S0}}{V_{P0}} = \frac{t_{P0}}{t_{S0}}. \quad (6)$$

In principle, the parameters $V_{\text{nmo},P}$, η , and $V_{\text{nmo},S}$ and the vertical times t_{P0} and t_{S0} are sufficient for recovering all four unknowns (V_{P0} , V_{S0} , ϵ , and δ). For example, δ can be found as

$$1 + 2\delta = \left(\frac{t_{P0}}{t_{S0}} \right)^2 \frac{1}{(V_{\text{nmo},S}/V_{\text{nmo},P})^2 - 2\eta}. \quad (7)$$

Then the parameters V_{P0} , ϵ , and V_{S0} can be obtained from equations (1), (2), and (6). This inversion procedure remains valid in multilayered media where the interval NMO velocities of PP- and SS-waves can be combined with the interval η values to perform parameter estimation.

However, as discussed by Tsvankin and Grechka (2000a), the inversion of the parameters $V_{\text{nmo},P}$, η , $V_{\text{nmo},S}$, and t_{P0}/t_{S0} is unstable and cannot be used in practice. Equation (7) gives additional insight into the reasons for the failure of this approach. In the presence of realistic errors in the NMO velocities and (in particular) the parameter η , the denominator $[(V_{\text{nmo},S}/V_{\text{nmo},P})^2 - 2\eta]$ may become small or even vanish, which leads to substantial amplification of measurement errors in the estimation of δ .

The influence of small errors in η on the values of δ obtained from equation (7) is illustrated by Figure 2. Deviations of η from the correct value of just ± 0.02 lead to unacceptable errors in δ reaching 0.2. Since η can seldom be estimated with accuracy higher than ± 0.05 – 0.1 (Alkhalifah, 1997; Grechka and Tsvankin, 1998), the corresponding errors in δ can far exceed those shown in Figure 2.

INVERSION OF PP MOVEOUT AND LONG-SPREAD PS DATA

Li and Yuan (1999) suggest overcoming the instability in resolving the vertical velocities and the parameters ϵ and δ by using PS moveout at large (compared with the reflector depth) offsets. Here, we demonstrate that the combination of long-spread moveouts of PP- and PS-waves is still insufficient for unambiguous parameter estimation.

Let us perturb a typical VTI model from Tables 1 and 2 (top row) by introducing small errors into the parameter $\eta = 0.136$. To keep the long-spread PP-wave moveout almost unchanged, the errors in η are compensated by small variations (up to 1%) in $V_{\text{nmo},P}$ (Grechka and Tsvankin, 1998), while the vertical traveltimes and the S-wave NMO velocity are fixed at the correct values. For each perturbed (erroneous) model we computed the vertical velocities, reflector depth, and parameters ϵ , δ , and σ using equations (1), (2), and (5)–(7) (Table 2).

As expected, the small variations in $V_{\text{nmo},P}$ and η lead to significant errors in the vertical velocities (greater than 10%), reflector depth D and the anisotropic parameters. However, since all four erroneous models have the correct vertical traveltimes and only a small error in the PS-wave NMO velocity, they produce practically identical PS-wave traveltimes for offsets limited by the reflector depth. Therefore, in agreement with the analysis above, the combination of long-spread PP-wave moveout and conventional-spread PS-wave moveout does not constrain the VTI parameters and reflector depth.

Next, we verify if it is possible to distinguish between the models listed in Table 2 using long-spread moveout of PS-waves. The PP- and PS-wave reflection traveltimes computed by anisotropic ray tracing for the correct and four inverted (erroneous) models are compared in Figure 3. The maximum offset in this test was fixed at $X = 3$ km, so the maximum offset-to-depth ratio X/D varies from 2.7 to 3.4 in accordance with the change in the reflector depth (Table 2). Clearly, all four vastly different erroneous models have virtually the same long-spread moveout of both PP- and PS-waves as the correct

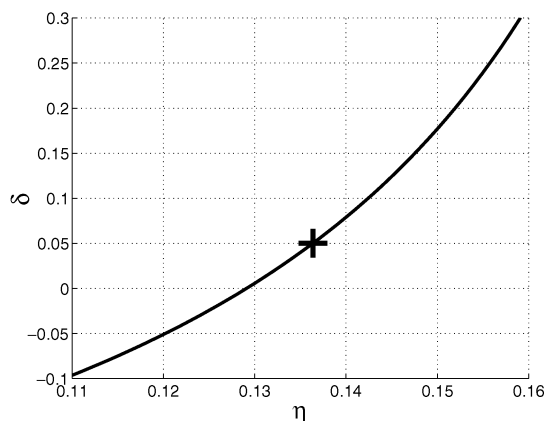


FIG. 2. The parameter δ computed from equation (7) for a range of η values and the exact parameters t_{P0} , t_{S0} , $V_{\text{nmo},P}$, and $V_{\text{nmo},S}$. The VTI model parameters are $V_{P0} = 2.5$ km/s, $V_{S0} = 1.0$ km/s, $\epsilon = 0.20$, $\delta = 0.05$; the layer thickness $D = 1.0$ km. The cross marks the correct values of η and δ . Note how small errors in η get amplified in the computation of δ .

model. The traveltime errors barely exceed 1 ms for models 1 and 4, and are even smaller for models 2 and 3.

Thus, even for large offset-to-depth ratios seldom attained in practice, the combination of PP and PS traveltimes cannot be inverted for the vertical velocities and the parameters ϵ and δ . Because of the limited range of reflection S-wave angles for mode conversions, the traveltimes of PS-waves at large offsets are primarily controlled by the S-wave NMO velocity and the parameter η (which can be estimated from long-spread PP traveltimes) rather than by ϵ and δ separately. It should be mentioned, however, that the increased incidence angle of P-waves for large-offset PS arrivals helps to put tighter constraints on η .

Table 1. Correct and erroneous sets of the moveout parameters. The four erroneous sets have slightly distorted values of $V_{\text{nmo},P}$ and η .

Data	t_{P0} (s)	t_{S0} (s)	$V_{\text{nmo},P}$ (km/s)	$V_{\text{nmo},S}$ (km/s)	η
Correct	0.800	2.000	2.622	1.696	0.136
Erroneous					
1	0.800	2.000	2.648	1.696	0.116
2	0.800	2.000	2.635	1.696	0.126
3	0.800	2.000	2.609	1.696	0.146
4	0.800	2.000	2.596	1.696	0.156

Table 2. Correct VTI parameters and those corresponding to the four erroneous sets of moveout parameters from Table 1.

Model parameters	V_{P0} (km/s)	V_{S0} (km/s)	ϵ	δ	σ	D (km)
Correct	2.500	1.000	0.200	0.050	0.938	1.000
Erroneous						
1	2.787	1.115	0.057	-0.049	0.657	1.115
2	2.646	1.058	0.121	-0.004	0.783	1.058
3	2.349	0.939	0.298	0.117	1.128	0.939
4	2.190	0.876	0.421	0.202	1.372	0.876

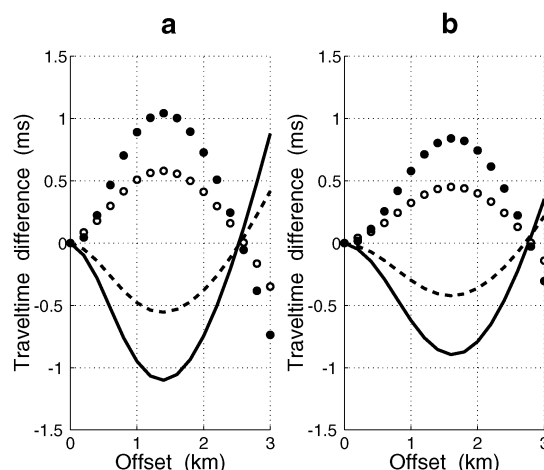


FIG. 3. Differences between the traveltimes of PP-waves (a) and PS-waves (b) computed for the erroneous models from Tables 1 and 2 and for the correct model. The solid line corresponds to model 1 from Table 2, the dashed line to model 4, the empty circles to model 3, and the filled circles to model 4.

DISCUSSION AND CONCLUSIONS

Recent progress in acquiring high-quality multicomponent data in ocean-bottom surveys opens new possibilities for anisotropic velocity analysis. For horizontally layered VTI media, supplementing PP-wave reflection data with traveltimes of mode-converted PS-waves on conventional-length spreads allows one to estimate the NMO velocity of pure SS-waves (which are not physically excited) and the ratio of the P and S vertical velocities. Although the combination of long-spread PP data and conventional-spread PS data can be formally inverted for the vertical velocities V_{P0} and V_{S0} and Thomsen coefficients ϵ and δ , we confirmed the result of Tsvankin and Grechka (2000a) that such an inversion is unstable and cannot be applied to field data.

Since mode conversions are known for the large magnitude of their nonhyperbolic moveout, it seems that including large-offset PS traveltimes should help to reduce this ambiguity. However, our analysis shows that long-spread PP and PS data in horizontally layered VTI media are still insufficient for recovering the VTI parameters and reflector depth. Numerical testing reveals an infinite number of models with vastly different parameters V_{P0} , V_{S0} , ϵ , and δ , which have practically the same reflection moveout of PP- and PS-waves for offsets as large as three times the reflector depth. Although all of those models are suitable for time processing, they have distorted vertical velocities and reflector depths. Stable depth-domain inversion for horizontally layered VTI media requires using the nonhyperbolic moveout of PP-waves and pure SS-waves (Tsvankin and Thomsen, 1995).

The joint inversion of PP and PS data in VTI media becomes much better posed in the presence of dipping reflectors. A 2-D parameter estimation on the dip line of the structure requires both horizontal and dipping PP and PS events in each depth interval, whereas 3-D inversion can be performed for wide-azimuth multicomponent data from a single mildly dipping interface (Tsvankin and Grechka, 2000a, b).

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