Moveout-based geometrical-spreading correction for PS-waves in layered anisotropic media

2008 J. Geophys. Eng. 5 195

(http://iopscience.iop.org/1742-2140/5/2/006)

IOP PUBLISHING JOURNAL OF GEOPHYSICS AND ENGINEERING
J. Geophys. Eng. **5** (2008) 195–202 doi:10.1088/1742-2132/5/2/006

Moveout-based geometrical-spreading correction for PS-waves in layered anisotropic media

Xiaoxia Xu[1,3] **and Ilya Tsvankin**[2]

[1] Formerly at Center for Wave Phenomena, Department of Geophysics, Colorado School of Mines, Golden, CO 80401-1887, USA
[2] Center for Wave Phenomena, Department of Geophysics, Colorado School of Mines, Golden, CO 80401-1887, USA

E-mail: ellen.xiaoxia.xu@exxonmobil.com and ilya@mines.edu

Received 19 November 2007
Accepted for publication 3 April 2008
Published 20 May 2008
Online at stacks.iop.org/JGE/5/195

Abstract
This paper is devoted to pre-stack amplitude analysis of reflection seismic data from anisotropic (e.g., fractured) media. Geometrical-spreading correction is an important component of amplitude-variation-with-offset (AVO) analysis, which provides high-resolution information for anisotropic parameter estimation and fracture characterization. Here, we extend the algorithm of moveout-based anisotropic spreading correction (MASC) to mode-converted PSV-waves in VTI (transversely isotropic with a vertical symmetry axis) media and symmetry planes of orthorhombic media. While the geometrical-spreading equation in terms of reflection traveltime has the same form for all wave modes in laterally homogeneous media, reflection moveout of PS-waves is more complicated than that of P-waves (e.g., it can become asymmetric in common-midpoint geometry). Still, for models with a horizontal symmetry plane, long-spread reflection traveltimes of PS waves can be well approximated by the Tsvankin–Thomsen and Alkhalifah–Tsvankin moveout equations, which are widely used for P-waves. Although the accuracy of the Alkhalifah–Tsvankin equation is somewhat lower, it includes fewer moveout parameters and helps to maintain the uniformity of the MASC algorithm for P- and PS-waves. The parameters of both moveout equations are obtained by least-squares traveltime fitting or semblance analysis and are different from those for P-waves. Testing on full-waveform synthetic data generated by the reflectivity method for layered VTI media confirms that MASC accurately reconstructs the plane-wave conversion coefficient from conventional-spread PS data. Errors in the estimated conversion coefficient, which become noticeable at moderate and large offsets, are mostly caused by the offset-dependent transmission loss of PS-waves.

Keywords: reflection seismology, anisotropic media, transverse isotropy, converted waves, amplitude variation with offset, nonhyperbolic moveout

(Some figures in this article are in colour only in the electronic version)

Introduction

Amplitude analysis is widely used in reflection seismology for purposes of hydrocarbon detection, lithology discrimination, etc. The main advantage of amplitude methods compared to traveltime inversion is their high vertical resolution, which makes amplitude-variation-with-offset (AVO) analysis applicable to relatively thin reservoirs. AVO inversion plays an important role in the presence of velocity anisotropy because it can provide essential information for anisotropic

[3] Present address: ExxonMobil Upstream Research Company, Houston, TX 77252, USA.

parameter estimation and fracture characterization (e.g., Rüger 2002).

In particular, the azimuthally varying AVO response measured on wide-azimuth data acquired above fractured reservoirs can help to estimate the dominant fracture direction and identify 'sweet spots' of intense fracturing (Rüger and Tsvankin 1997, Neves *et al* 2003). However, inversion of the P-wave AVO response for the pertinent anisotropy parameters is generally ambiguous even for the simple HTI (transversely isotropic with a horizontal symmetry axis) medium formed by a system of vertical, penny-shaped cracks in isotropic host rock (Rüger and Tsvankin 1997, Bakulin *et al* 2000a, Rüger 2002). Furthermore, adequate description of most fractured formations requires using lower (i.e., more complicated) orthorhombic symmetry (Bakulin *et al* 2000b, Grechka *et al* 2005).

In principle, the nonuniqueness of AVO analysis can be overcome by combining the P-wave AVO gradient determined from the initial slope of the reflection coefficient with the normal-moveout (NMO) ellipse (e.g., Bakulin *et al* 2000a), but this approach has serious limitations. First, the NMO ellipse can be reconstructed only for relatively thick reservoirs; second, the difference in vertical resolution between amplitude and traveltime methods can lead to distorted estimates for heterogeneous reservoir formations (Xu and Tsvankin 2007).

For surveys with multicomponent acquisition, the AVO response of P-waves can be supplemented with that of mode-converted PS-waves. The main advantage of combining P and PS amplitude signatures is that they are determined by rock properties on the same scale near the top or bottom of the reservoir. Bakulin *et al* (2000a) showed that the azimuthally varying AVO gradients of P- and PS-waves reflected from an HTI medium constrain both the normal and tangential compliances of the fractures. The compliances can then be related to such physical properties as fracture density and fluid infill. Although this technique was introduced for boundaries between isotropic and HTI media, it remains valid for the lower symmetry orthorhombic model that describes a vertical fracture system in a VTI background matrix (Bakulin *et al* 2000b). A more general methodology for the joint inversion of the long-offset, wide-azimuth AVO responses of P-waves and split PS-waves in azimuthally anisotropic media was developed by Jílek (2002).

Since shear-wave (and, therefore, converted-wave) amplitudes are highly sensitive to the presence of anisotropy along the raypath, robust estimation of PS-wave reflection (conversion) coefficients for the target horizon is impossible without an accurate geometrical-spreading correction. As discussed by Tsvankin (1995, 2005) and Xu *et al* (2005), geometrical spreading of SV-waves in TI media is controlled primarily by the parameter $\sigma \equiv \left(V_{P0}^2/V_{S0}^2\right)(\epsilon - \delta)$, which is typically much larger than the Thomsen parameters ϵ and δ responsible for P-wave amplitudes (V_{P0} and V_{S0} are the symmetry-direction P- and S-wave velocities, respectively).

To correct AVO signatures for amplitude distortions in the overburden, it is convenient to represent geometrical spreading through reflection traveltimes. Following paraxial ray theory, Červený (2001) and Xu *et al* (2005) obtained geometrical spreading of pure reflection modes (P or S) in layered, arbitrarily anisotropic media as a function of traveltime derivatives. Although this equation is strictly valid only for laterally homogeneous models, it remains sufficiently accurate in the presence of moderate dips and mild lateral velocity variation (Xu 2006).

By combining this geometrical-spreading formulation with a 3D extension of the Alkhalifah–Tsvankin (1995) nonhyperbolic moveout equation, Xu and Tsvankin (2006a) developed a practical and robust algorithm for moveout-based anisotropic spreading correction ('MASC'). They expressed the traveltime derivatives needed in the geometrical-spreading computation through the moveout coefficients, which are estimated by nonhyperbolic semblance analysis. The accuracy of MASC for wide-azimuth, long-spread P-wave data from layered orthorhombic media was confirmed by dynamic ray tracing and full-waveform synthetic modelling (Xu and Tsvankin 2006b). The synthetic tests demonstrate that if the azimuthal variation of geometrical spreading is not negligible, MASC cannot be replaced by empirical gain corrections even in qualitative AVO analysis. The method was also successfully applied to azimuthal AVO analysis of P-wave data acquired above a fractured reservoir at Rulison field in Colorado, USA (Xu and Tsvankin 2007).

Here, the methodology of MASC is applied to PS-waves converted at the reflector (so-called 'C-waves') in laterally homogeneous, anisotropic media. First, we discuss the equivalence of traveltime-based geometrical-spreading equations for converted and pure modes. Second, by employing the Tsvankin–Thomsen (1994) and Alkhalifah–Tsvankin (1995) moveout equations, the MASC algorithm is implemented for PSV-waves acquired in vertical symmetry planes of layered TI and orthorhombic media. Finally, we conduct a full-waveform synthetic study to evaluate the accuracy of MASC in estimating the conversion coefficient and compare its performance with that of empirical gain corrections used in practice.

Moveout-based geometrical-spreading equation for PS-waves

One of the most significant differences between P and PS reflected events is the asymmetry of the raypath and moveout of mode conversions. If the medium is laterally heterogeneous or anisotropic without a horizontal symmetry plane, the traveltime of PS-waves does not remain the same when the source and receiver are interchanged (Thomsen 1999, Tsvankin and Grechka 2000, Dewangan 2004). Because of this moveout asymmetry, the PS-wave reflection traveltime on common-midpoint (CMP) gathers may not be an even function of offset and cannot be described by conventional moveout equations for P-waves. Therefore, the two key components of MASC (i.e., the geometrical-spreading and moveout equations) have to be revisited for converted waves.

The general traveltime-based expression for geometrical spreading of pure modes is derived in appendix A of Xu *et al* (2005). Although the derivation assumes reflection moveout to be symmetric (i.e., independent of the sign of offset), the final result is valid for converted waves. The ray-theory representation of geometrical spreading at the earth's surface (Červený 2001) includes traveltime derivatives with respect to four variables—the horizontal coordinates of the source and the receiver. If the medium is laterally homogeneous, the number of independent variables can be reduced to two (offset x and azimuth α), even for arbitrary anisotropic symmetries. For pure reflection modes, the azimuth varies only from 0° to 180° because their traveltime remains the same when the source and receiver are interchanged.

The only modification required to account for the asymmetric moveout of converted waves is extension of the range of azimuths to 360°. Then the definition of azimuth (equation (A-3) of Xu *et al* 2005) takes the form

$$\alpha = \tan^{-1}\left[\frac{x_2^r - x_2^s}{x_1^r - x_1^s}\right] \qquad \left(x_1^r - x_1^s > 0\right), \tag{1}$$

$$\alpha = \tan^{-1}\left[\frac{x_2^r - x_2^s}{x_1^r - x_1^s}\right] + \pi \qquad \left(x_1^r - x_1^s < 0\right), \tag{2}$$

where $x_{1,2}^s$ and $x_{1,2}^r$ are the horizontal source and receiver coordinates, respectively. Compared to the original definition for pure modes, equation (2) contains an additional constant (π), which does not change the traveltime derivatives.

Hence, the moveout-based geometrical-spreading equation given by Xu *et al* (2005) is entirely valid for converted waves:

$$L(x,\alpha) = (\cos\phi^s \cos\phi^r)^{1/2} \times \left[\frac{\partial^2 T}{\partial x^2}\frac{\partial T}{\partial x}\frac{1}{x} + \frac{\partial^2 T}{\partial x^2}\frac{\partial^2 T}{\partial \alpha^2}\frac{1}{x^2} - \left(\frac{\partial T}{\partial \alpha}\right)^2 \frac{1}{x^4}\right]^{-1/2}, \tag{3}$$

where T is the traveltime, and ϕ^s and ϕ^r are the angles between the ray and the vertical at the source and receiver locations, respectively. Equation (3) can be used for any reflected wave (pure or converted) in laterally homogeneous, arbitrarily anisotropic media. Application of this equation to events with asymmetric moveout, however, requires certain care because the traveltime derivatives are different for 'reciprocal' source–receiver pairs with azimuths $\alpha \pm \pi$.

Note that the only source of moveout asymmetry for converted waves in laterally homogeneous media is the presence of anisotropic layers that do not have a horizontal symmetry plane. Next, we verify that our formalism accurately describes the geometrical spreading of PS-waves in the simplest model of this type, which includes a homogeneous TI layer with a tilted symmetry axis (TTI; see figure 1). The TTI parameters used in our test are taken from the physical model of Dewangan *et al* (2006).

Figure 1 shows the traveltime surface of the fast PS-wave (i.e., wave PS_1) computed by anisotropic ray tracing. In the vertical plane that contains the symmetry axis (called the 'symmetry-axis plane'), the fast S-wave has in-plane polarization and, therefore, represents the SV mode. Since the horizontal plane in this model is not a plane of symmetry, the traveltime surface in the CMP geometry is asymmetric with respect to the global minimum. Also, the minimum traveltime is shifted from the common midpoint to an offset that exceeds 0.5 km.

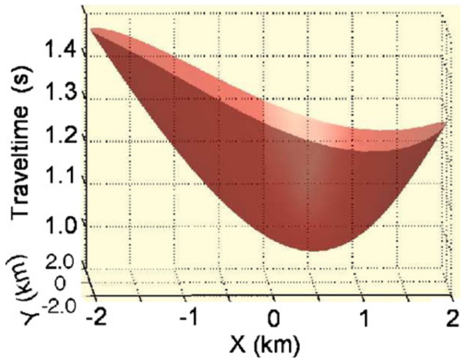

Figure 1. Traveltime surface of the fast PS-wave computed for a horizontal TTI layer in the common-midpoint (CMP) geometry. The model parameters are $V_{P0} = 2.6$ km s^{-1}, $V_{S0} = 1.38$ km s^{-1}, $\epsilon = 0.46$, $\delta = 0.11$ and $\gamma = 0$. The tilt of the symmetry axis from the vertical is $\nu = 70°$, the layer's thickness is 1 km. Note the asymmetry of the surface with respect to the global traveltime minimum, which does not correspond to zero offset.

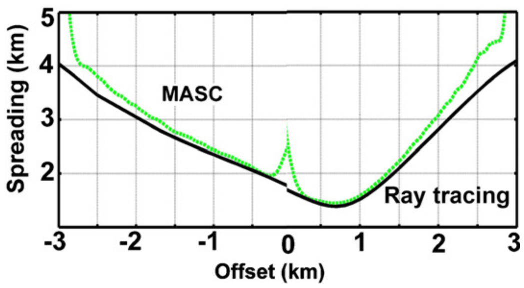

Figure 2. Comparison of the PS-wave geometrical spreading computed from equation (3) (dashed line) and ray tracing (solid line) in the symmetry-axis plane of the model in figure 1. The 'jitters' in the output of MASC are caused by local errors in approximating the traveltime surface.

Because it is difficult to approximate this traveltime surface with a Taylor series, we employed a cubic-spline function. Substituting the traveltime derivatives obtained from this function into equation (3), we computed the spreading for the PSV-wave in the symmetry-axis plane. Comparison with dynamic ray tracing in figure 2 confirms the accuracy of equation (3) for converted waves with asymmetric moveout. Except for the discrepancies at large $x \to \pm 3$ km and zero offset, which are caused by numerical problems in estimating the second-order traveltime derivatives, the spreading computed by our method is close to that obtained by ray tracing.

Although this test shows that MASC can handle asymmetric moveout functions, the rest of the paper is focused on models with a horizontal symmetry plane, in which PS-wave moveout is symmetric.

MASC algorithm for PS-waves

Outside the symmetry planes of azimuthally anisotropic media, a P-wave incident upon a horizontal reflector excites two split PS-waves which have to be separated using polarization analysis. To avoid this complication and facilitate AVO processing, we assume that the acquisition line is confined to a vertical symmetry plane of the model. Then a P-wave source generates only a P-to-SV conversion polarized in the incidence plane. The goal of this section is to extend the MASC methodology of Xu and Tsvankin (2006a) to PSV-waves recorded in vertical symmetry planes of horizontally layered VTI, HTI and orthorhombic media. Note that equation (3) includes derivatives of the traveltime with respect to azimuth, which are determined by traveltime variations outside the incidence plane.

Estimation of the group angles ϕ^s and ϕ^r from surface reflection data requires knowledge of the velocity in the subsurface layer. Following the approach suggested by Xu and Tsvankin (2006a) for pure modes, first we compute the time slopes (horizontal slownesses) on common-shot and common-receiver gathers of the PS-wave. Then the angles ϕ^s and ϕ^r are obtained from the horizontal slownesses under the assumption that the subsurface layer is locally isotropic near the source and receiver locations.

As is the case for P-waves, the key issue in implementing equation (3) for mode conversions is to find a smooth, relatively simple traveltime approximation that can be used for a wide range of offsets and azimuths. Long-spread reflection moveout of P-waves in layered VTI media is well described by the Tsvankin–Thomsen (1994) nonhyperbolic equation:

$$T^2(x) = T_0^2 + A_2 x^2 + \frac{A_4 x^4}{1 + A x^2}, \tag{4}$$

where T_0 is the zero-offset time, $A_2 = V_{\rm nmo}^{-2}$ controls hyperbolic moveout ($V_{\rm nmo}$ is the normal-moveout velocity), and A_4 is the quartic coefficient responsible for nonhyperbolic moveout at large offsets. The parameter A depends on the horizontal velocity and is introduced to make $T(x)$ convergent at $x \to \infty$. With an appropriate substitution of the moveout parameters, equation (4) gives sufficient accuracy for PS-wave traveltimes in horizontally layered VTI media (Tsvankin 2005).

By taking into account the azimuthal variation of the moveout parameters A_2, A_4 and A, Al-Dajani *et al* (1998) extended equation (4) to P-waves in orthorhombic media:

$$T^2(x,\alpha) = T_0^2 + A_2(\alpha) x^2 + \frac{A_4(\alpha) x^4}{1 + A(\alpha) x^2}; \tag{5}$$

$$A_2(\alpha) = A_2^{(1)} \sin^2\alpha + A_2^{(2)} \cos^2\alpha, \tag{6}$$

$$A_4(\alpha) = A_4^{(1)} \sin^4\alpha + A_4^{(2)} \cos^4\alpha + A_4^{(x)} \sin^2\alpha \cos^2\alpha. \tag{7}$$

The dependence of A_2 on the azimuth α is described by the NMO ellipse (Grechka and Tsvankin 1998), and equation (7) for A_4 is derived by Al-Dajani *et al* (1998) for a horizontal orthorhombic layer. (Note that HTI can be treated as a special case of the more general orthorhombic model.) It is assumed in equations (5)–(7) that $\alpha = 0$ corresponds to the symmetry plane $[x_1, x_3]$, so $A_2^{(1,2)}$ and $A_4^{(1,2)}$ are the symmetry-plane moveout coefficients, while $A_4^{(x)}$ contributes to nonhyperbolic moveout in off-symmetry directions. Because of the difficulties in treating split PS-waves outside the symmetry planes, equation (5) has not been applied to mode conversions.

Table 1. Parameters of a medium that includes a VTI layer sandwiched between two isotropic layers (model 1). The velocities and anisotropy parameters of the VTI layer are taken from the measurements for Dog Creek shale listed in Thomsen (1986).

	Layer 1	Layer 2	Layer 3
Symmetry type	ISO	VTI	ISO
Thickness (km)	0.5	1.0	∞
Density (g cm^{-3})	2.0	2.1	2.2
V_{P0} (km s^{-1})	1.7	2.2	2.2
V_{S0} (km s^{-1})	0.8	1.1	1.0
ϵ	0	0.23	0
δ	0	0.10	0
γ	0	0.10	0
η	0	0.10	0
σ	0	0.64	0

Alkhalifah and Tsvankin (1995) proposed a simpler nonhyperbolic moveout equation for P-waves in VTI media which depends on only two parameters, the velocity $V_{\rm nmo}$ and anellipticity coefficient $\eta \equiv (\epsilon - \delta)/(1 + 2\delta)$:

$$T^2(x) = T_0^2 + \frac{x^2}{V_{\rm nmo}^2} - \frac{2\eta x^4}{V_{\rm nmo}^2\left[T_0^2 V_{\rm nmo}^2 + (1+2\eta)x^2\right]}. \tag{8}$$

Equation (8) is widely used to correct long-spread data for nonhyperbolic moveout and estimate the key anisotropy parameter η, which is responsible for time processing of P-wave data in VTI media. The 3D version of the Alkhalifah–Tsvankin equation provides a close approximation to wide-azimuth P-wave traveltimes in orthorhombic or HTI media (Vasconcelos and Tsvankin 2006):

$$T^2(x,\alpha) = T_0^2 + \frac{x^2}{V_{\rm nmo}^2(\alpha)} - \frac{2\eta(\alpha) x^4}{V_{\rm nmo}^2(\alpha)\left[T_0^2 V_{\rm nmo}^2(\alpha) + (1+2\eta(\alpha))x^2\right]}, \tag{9}$$

$$V_{\rm nmo}^{-2}(\alpha) = \frac{\sin^2\alpha}{\left(V_{\rm nmo}^{(1)}\right)^2} + \frac{\cos^2\alpha}{\left(V_{\rm nmo}^{(2)}\right)^2}, \tag{10}$$

$$\eta(\alpha) = \eta^{(1)} \sin^2\alpha + \eta^{(2)} \cos^2\alpha - \eta^{(3)} \sin^2\alpha \cos^2\alpha. \tag{11}$$

Equation (10), which is equivalent to equation (6) discussed above, describes the NMO ellipse with the semi-axes $V_{\rm nmo}^{(1)}$ and $V_{\rm nmo}^{(2)}$; $\eta^{(1,2,3)}$ are the anellipticity parameters in the three mutually orthogonal symmetry planes of the model. Xu and Tsvankin (2006a, 2006b) employed equations (9)–(11) to compute the geometrical spreading of P-waves in horizontally layered, azimuthally anisotropic media from equation (3).

Although equations (8) and (9) were originally designed for P-waves, it is worthwhile to test them for PS-waves. While the analytic form of the moveout parameters is not expected to be the same for P- and PS-waves, the geometrical-spreading correction operates with the coefficients obtained from semblance analysis or traveltime fitting (if traveltimes have been picked). Therefore, we need to verify if equations (8) and (9) with the *best-fit* parameters $\hat{V}_{\rm nmo}$ and $\hat{\eta}$ provide sufficient accuracy for long-spread converted-wave moveout.

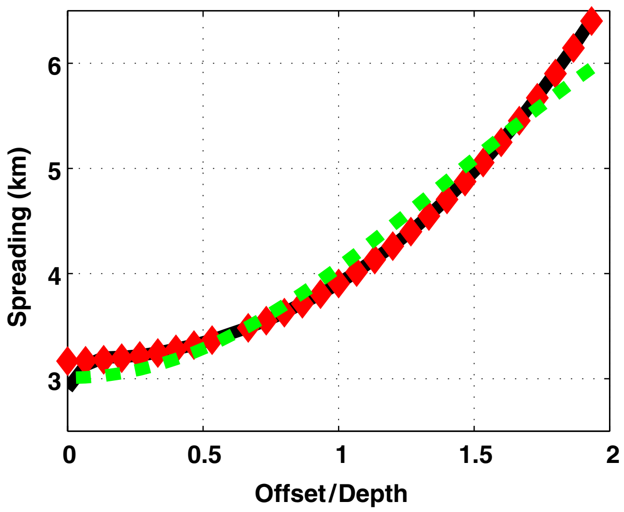

Figure 3. Geometrical spreading of PSV-waves reflected from the bottom of the VTI layer in model 1 (table 1). Our method was applied with the Tsvankin–Thomsen equation (4) (diamonds) and with the Alkhalifah–Tsvankin equation (8) (dashed line); the solid line is computed by dynamic ray-tracing code ANRAY (Gajewski and Pšenčík 1987).

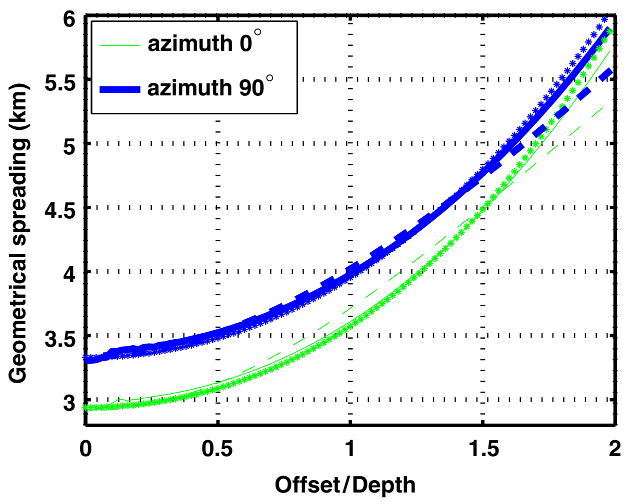

Figure 4. Geometrical spreading of PSV-waves reflected from the bottom of the orthorhombic layer in model 2 (table 2). The azimuths $\alpha = 0°$ and $\alpha = 90°$ correspond to the symmetry planes $[x_1, x_3]$ and $[x_2, x_3]$, respectively. Our method was applied with the 3D Tsvankin–Thomsen equation (5) (stars) and with the 3D Alkhalifah–Tsvankin equation (9) (dashed lines); the solid lines are computed by dynamic ray tracing.

First, we conducted a test for a PSV reflection from the bottom of a VTI layer sandwiched between two isotropic half-spaces (figure 3 and table 1). After computing exact traveltimes of the PS-wave by anisotropic ray tracing, we applied the least-squares method to estimate the parameters of equations (4) and (8). The best-fit moveout parameters were then substituted into the geometrical-spreading equation (3). In agreement with the results of Tsvankin (2005), equation (4) provides an excellent approximation for PSV-wave moveout and yields a geometrical-spreading factor that is almost identical to that computed by dynamic ray tracing (figure 3). Although the performance of the Alkhalifah–Tsvankin equation (8) is somewhat inferior, it has the advantage of being consistent with the P-wave formalism while still providing adequate accuracy. It should be mentioned that the best-fit parameter $\hat{\eta}$ for PSV-waves is different from its analytic definition ($\eta \equiv (\epsilon - \delta)/(1 + 2\delta)$ in a single VTI layer) for P-waves.

Next, the moveout approximations and the MASC methodology were applied to a model that includes an orthorhombic layer (figure 4 and table 2). Note that P-waves are coupled to two different split S-waves in the symmetry planes $[x_1, x_3]$ and $[x_2, x_3]$ (Tsvankin 2005). Indeed, if the fast shear wave S_1 is polarized in the x_1-direction at vertical incidence, it represents the SV mode (which will produce P-to-SV conversion) in the $[x_1, x_3]$-plane. Then the slow shear wave S_2 will be responsible for the converted PSV-wave in the $[x_2, x_3]$-plane.

Since the symmetry planes of orthorhombic models are kinematically equivalent to VTI, the traveltime fit provided by the moveout approximations in the incidence plane is the same as in VTI media. Geometrical spreading in azimuthally anisotropic media (equation (3)), however, also depends on azimuthal traveltime variations away from the incidence plane (Tsvankin 2005, Xu *et al* 2005). Therefore, the accuracy of our method depends on the performance of the 3D versions of the moveout equations in the vicinity of the symmetry planes. As was the case for VTI media, the error of our method with the 3D Tsvankin–Thomsen equation (5) is almost negligible, while the 3D Alkhalifah–

Table 2. Parameters of a medium composed of VTI, orthorhombic and isotropic layers (model 2). Orthorhombic symmetry can be described by the two vertical velocities (V_{P0} for P-waves and V_{S0} for the S-wave polarized in the x_1-direction) and seven anisotropy parameters ($\epsilon^{(1)}$, $\epsilon^{(2)}$, $\delta^{(1)}$, $\delta^{(2)}$, $\delta^{(3)}$, $\gamma^{(1)}$ and $\gamma^{(2)}$); the parameter values are based on the measurements of Wang (2002). The anellipticity parameters $\eta^{(1)}$, $\eta^{(2)}$, $\eta^{(2)}$ control P-wave nonhyperbolic moveout, while $\sigma^{(1)}$ and $\sigma^{(2)}$ are largely responsible for the moveout of SV-waves in the vertical symmetry planes. For a detailed explanation of the notation, see Tsvankin (2005).

	Layer 1	Layer 2	Layer 3
Symmetry type	VTI	ORTH	ISO
Thickness (km)	0.5	1.0	∞
Density (g cm^{-3})	2.1	2.1	2.2
V_{P0} (km s^{-1})	2.2	2.2	2.2
V_{S0} (km s^{-1})	1.1	1.1	1.0
$\epsilon^{(1)}$	0.23	0.317	0
$\delta^{(1)}$	0.10	−0.054	0
$\gamma^{(1)}$	0.10	0.513	0
$\epsilon^{(2)}$	0.23	0.121	0
$\delta^{(2)}$	0.10	0.046	0
$\gamma^{(2)}$	0.10	0.138	0
$\delta^{(3)}$	0	0.1	0
$\eta^{(1)}$	0.1	0.42	0
$\eta^{(2)}$	0.1	0.07	0
$\eta^{(3)}$	0	0.05	0
$\sigma^{(1)}$	0.64	1.48	0
$\sigma^{(2)}$	0.64	0.31	0

Tsvankin equation (9) produces some deviations from the ray-tracing result, especially at far offsets (figure 4). Still, given relatively large uncertainty in amplitude measurements, the accuracy of equation (9) should be acceptable for purposes of AVO analysis.

Application to AVO analysis of synthetic data

In addition to potential problems with moveout approximations, the accuracy of amplitude corrections designed to estimate the reflection coefficient may be influenced by several other factors. Here, we do not consider PS-wave amplitudes in the anomalous areas near shear-wave cusps (triplications) and singularities, where AVO analysis is not practical. Still, the high sensitivity of S-waves to the presence of anisotropy may lead to rapid amplitude variations along PS wave fronts that are not adequately described by ray theory and, therefore, by MASC (Tsvankin 2005). Also, even for models with a horizontal symmetry plane, the asymmetry of the PS raypath (i.e., the difference between the P- and S-legs) can result in a significant angular variation of transmission loss and related errors in AVO analysis. Hence, it is essential to test the performance of MASC for PS-waves on 3D full-waveform synthetic data, as was done by Xu and Tsvankin (2006b) for P-waves.

The main question to be answered in this section is how accurately MASC can reconstruct plane-wave conversion coefficients in layered anisotropic media. In particular, we evaluate the magnitude of transmission loss (which is not included in MASC) and the related distortions of the PS-wave AVO response. Also, an important practical issue is whether or not MASC can be replaced by empirical gain corrections in qualitative AVO analysis.

Due to the difficulties in modelling exact PS-wave amplitudes for layered orthorhombic media, we carried out amplitude processing only for model 1 (table 1) composed of isotropic and VTI layers. Synthetic seismograms were computed with the reflectivity code (ANISYNPA), which generates exact 3D wave fields for horizontally layered anisotropic media (e.g., Fryer and Frazer 1984). A shot gather of the vertical displacement from a vertical force for model 1 is shown in figure 5. The processing sequence is similar to that for P-waves described by Xu and Tsvankin (2006b). First, we apply the nonhyperbolic moveout equation (4) to the PS reflection from the bottom of the VTI layer to estimate the parameters A_2, A_4 and A, which serve as the input to the geometrical-spreading correction. To emulate processing of field data, the parameters were obtained from nonhyperbolic semblance analysis described by Vasconcelos and Tsvankin (2006) and Xu and Tsvankin (2006a). Second, the raw amplitudes are picked along the traveltime curve defined by equation (4) with the best-fit parameters. Third, MASC (equation (3)) is employed to correct the picked amplitudes for anisotropic geometrical spreading. Fourth, the source and receiver directivity factors are removed using local estimates of the horizontal slowness.

To calibrate the P-wave AVO response, we matched the corrected amplitude at normal incidence with the exact reflection coefficient (Xu and Tsvankin 2006b). This approach is not suitable for PS-waves because the conversion coefficient at normal incidence for the model at hand goes to zero. Since the source radiation factor should be the same for both P- and PS-waves, we calibrated PS-wave amplitudes using the scaling factor estimated for the corresponding P-wave reflection.

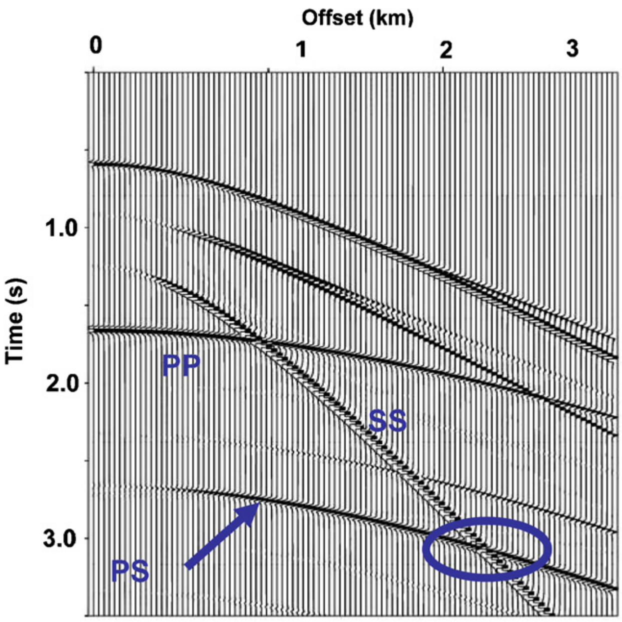

Figure 5. Synthetic shot gather for model 1 (table 1) computed by the anisotropic reflectivity method. The top layer is specified as a half-space to eliminate the influence of the free surface. The arrow marks the target PS reflection converted at the bottom of the VTI layer. The ellipse highlights the area of interference between the target event and the SS reflection from the top of the VTI layer.

The high accuracy of MASC with the Tsvankin–Thomsen moveout equation for the model in figure 5 was confirmed by the test in the previous section (see figure 3). Still, the VTI layer has a significant value of the parameter σ, which is primarily responsible for SV-wave velocity anisotropy and angle-dependent geometrical spreading. Strong amplitude variations along the wave front of the PS-wave may cause errors in the ray-theory equations employed in our method. Nevertheless, the conversion coefficient estimated by MASC is close to the exact values for a relatively wide range of horizontal slownesses (figure 6).

In contrast, application of the conventional t-gain correction results in unacceptable errors for the whole offset range. (The accuracy of the t^2-gain correction, not shown here, is even lower.) Clearly, anisotropy significantly distorts geometrical spreading of PS-waves in typical TI models. Hence, AVO analysis for converted waves cannot be implemented without a robust anisotropic spreading correction.

The conversion coefficient reconstructed by MASC deviates from the exact values with increasing offset. This deviation is caused by the combined influence of the transmission loss and interference of the target PS event with the SS reflection from the top of the VTI layer (see the ellipse

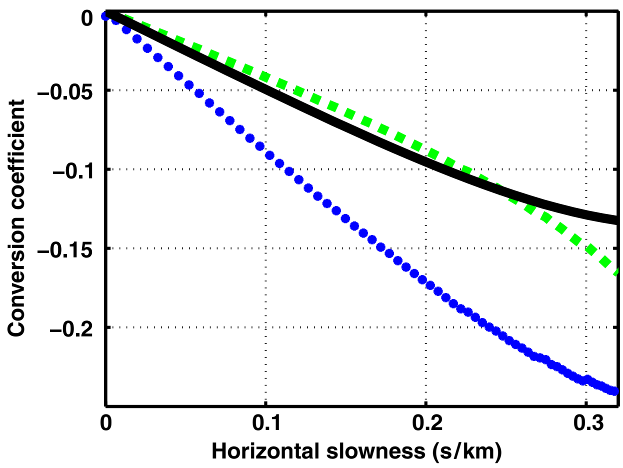

Figure 6. Conversion coefficient at the bottom of the VTI layer in model 1. The estimates obtained with MASC (dashed line) and the *t*-gain correction (dotted) are compared with the exact conversion coefficient (solid). The incidence angle of the downgoing P-wave corresponding to the maximum horizontal slowness (0.3 s km^{-1}) is 30°; for the upgoing SV-wave, the corresponding reflection angle is 15°.

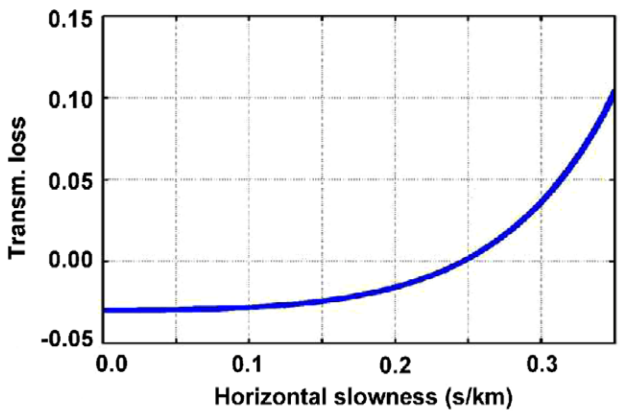

Figure 7. Transmission loss for the target PS-wave from figure 5. The loss is computed by subtracting from unity the product of the plane-wave transmission coefficients along the raypath.

in figure 5). Amplitude distortions caused by the interference with the SS-wave become especially severe for horizontal slownesses exceeding 0.3 s km^{-1}, which forced us to restrict the slowness range used in figure 6.

The transmission coefficients for the upgoing and downgoing segments of reflected P-rays compensate for each other in such a way that their product (which determines transmission loss) is almost invariant with the offset. For mode conversions, however, the upgoing and downgoing ray segments correspond to different modes, and this raypath asymmetry leads to an increase of the transmission loss for our model with the offset (figure 7). Since the geometrical-spreading correction does not account for transmission coefficients, this offset-dependent transmission loss distorts the reconstructed conversion coefficient in figure 6.

Discussion and conclusions

Amplitude-variation-with-offset (AVO) analysis, which is designed to operate with the plane-wave reflection coefficient at the target horizon, has to include a robust correction for geometrical spreading in the overburden. Geometrical spreading of shear and mode-converted waves typically is more strongly distorted by anisotropy than that of P-waves. Here, we showed that the moveout-based anisotropic spreading correction (MASC), previously developed for P-wave reflections, can be applied to PS-waves as well. For horizontally layered models, the geometrical-spreading factor of P- and PS-waves can be obtained from the same equation that involves the group (ray) angles at the surface and traveltime derivatives with respect to the offset and the azimuth. This equation remains valid even for models without a horizontal symmetry plane, such as tilted transverse isotropy, in which reflection moveout of PS-waves becomes asymmetric (i.e., traveltime does not stay the same when the source and receiver are interchanged).

Because of the difficulty in dealing with split PS-waves in azimuthally anisotropic media, our implementation of MASC for mode conversions is restricted to VTI media and symmetry planes of orthorhombic and HTI media. To compute the traveltime derivatives required by MASC, we employed the Tsvankin–Thomsen nonhyperbolic moveout equation, which is used almost exclusively for P-waves. Still, numerical testing proves that this equation gives a close approximation for PSV-wave moveout both in layered VTI media and in the vicinity of the vertical symmetry planes of orthorhombic media. The best-fit parameters of the Tsvankin–Thomsen equation serve as the input to the geometrical-spreading computation. Comparison with dynamic ray tracing shows that the accuracy of MASC for PS-waves is almost as high as that for P-waves.

Furthermore, for purposes of geometrical-spreading correction PS-wave traveltimes can be adequately described by the simpler Alkhalifah–Tsvankin equation[4]. Whereas the analytic form of that equation is valid only for P-waves, it can be applied to mode conversions with fitted moveout parameters. The Alkhalifah–Tsvankin equation has the important advantage of making the MASC algorithm for PS-waves fully consistent with that for P-waves at the expense of a somewhat lower quality of the traveltime fit.

Application of MASC to full-waveform synthetic data from layered VTI media yields accurate estimates of the plane-wave conversion coefficients for conventional-length spreads. The main complication in the reconstruction of conversion coefficients from surface data is caused by offset-dependent transmission loss of PS-waves. The product of the transmission coefficients along the asymmetric PS-wave raypath varies with incidence angle and, therefore, with offset. This variation, which is almost negligible for P-waves, is not accounted for in the geometrical-spreading correction and can produce significant distortions of the AVO response at moderate and large offsets.

[4] The Alkhalifah–Tsvankin equation includes one less parameter than the Tsvankin–Thomsen equation.

Our results demonstrate that even qualitative AVO analysis of PS-waves in the presence of anisotropic overburden requires application of the moveout-based anisotropic spreading correction. An important direction for future studies is to extend MASC to split PS-waves outside the symmetry planes of azimuthally anisotropic media. Such an extension is essential for developing robust AVO algorithms operating with wide-azimuth mode-converted data.

Acknowledgments

We are grateful to members of the A(nisotropy)-Team of the Center for Wave Phenomena (CWP), Colorado School of Mines, for helpful discussions. Constructive suggestions by the reviewers helped to improve the paper. The support for this work was provided by the Consortium Project on Seismic Inverse Methods for Complex Structures at CWP and by the Chemical Sciences, Geosciences and Biosciences Division, Office of Basic Energy Sciences, Office of Science, US Department of Energy.

References

Al-Dajani A, Tsvankin I and Toksöz M N 1998 Nonhyperbolic reflection moveout for azimuthally anisotropic media *68th Ann. Int. SEG Meeting* Expanded Abstracts pp 1479–82

Alkhalifah T and Tsvankin I 1995 Velocity analysis for transversely isotropic media *Geophysics* **60** 1550–66

Bakulin A, Grechka V and Tsvankin I 2000a Estimation of fracture parameters from reflection seismic data: part I. HTI model due to a single fracture set *Geophysics* **65** 1788–802

Bakulin A, Grechka V and Tsvankin I 2000b Estimation of fracture parameters from reflection seismic data: part II. Fractured models with orthorhombic symmetry *Geophysics* **65** 1803–17

Červený V 2001 *Seismic Ray Theory* (Cambridge: Cambridge University Press)

Dewangan P 2004 Processing and inversion of mode-converted waves using the PP+PS=SS method *PhD Thesis*, Colorado School of Mines

Dewangan P, Tsvankin I, Batzle M, van Wijk K and Haney M 2006 PS-wave moveout inversion for tilted TI media: a physical-modeling study *Geophysics* **71** D135–43

Fryer G J and Frazer L N 1984 Seismic waves in stratified anisotropic media *Geophys. J. R. Astron. Soc.* **78** 691–710

Gajewski D and Pšenčík I 1987 Computation of high frequency seismic wave fields in 3D laterally inhomogeneous anisotropic media *Geophys. J. R. Astron. Soc.* **91** 383–412

Grechka V, Pech A and Tsvankin I 2005 Parameter estimation in orthorhombic media using multicomponent wide-azimuth reflection data *Geophysics* **70** D1–8

Grechka V and Tsvankin I 1998 3D description of normal moveout in anisotropic inhomogeneous media *Geophysics* **63** 1079–92

Jílek P 2002 Modeling and inversion of converted-wave reflection coefficients in anisotropic media: a tool for quantitative AVO analysis: *PhD Thesis*, Colorado School of Mines

Neves F A, Al-Marzoug A, Kim J J and Nebrija E L 2003 Fracture characterization of deep tight sands using azimuthal velocity and AVO seismic data in Saudi Arabia *The Leading Edge* **22** 469–75

Rüger A 2002 *Reflection Coefficients and Azimuthal AVO Analysis in Anisotropic Media* (Soc. Explor. Geophys.)

Rüger A and Tsvankin I 1997 Using AVO for fracture detection: analytic basis and practical solutions *The Leading Edge* **16** 1429–34

Thomsen L 1986 Weak elastic anisotropy *Geophysics* **51** 1954–66

Thomsen L 1999 Converted-wave reflection seismology over inhomogeneous, anisotropic media *Geophysics* **64** 678–90

Tsvankin I 1995 Body-wave radiation patterns and AVO in transversely isotropic media *Geophysics* **60** 1409–25

Tsvankin I 2005 *Seismic Signatures and Analysis of Reflection Data in Anisotropic Media* 2nd edn (Amsterdam: Elsevier)

Tsvankin I and Grechka V 2000 Dip moveout of converted waves and parameter estimation in transversely isotropic media *Geophys. Prospect.* **48** 257–92

Tsvankin I and Thomsen L 1994 Nonhyperbolic reflection moveout in anisotropic media *Geophysics* **59** 1290–304

Vasconcelos I and Tsvankin I 2006 Non-hyperbolic moveout inversion of wide-azimuth P-wave data for orthorhombic media *Geophys. Prospect.* **54** 535–52

Wang Z 2002 Seismic anisotropy in sedimentary rocks: part 2. Laboratory data *Geophysics* **67** 1423–40

Xu X 2006 Anisotropic geometrical-spreading correction and its application in azimuthal AVO analysis *PhD Thesis*, Colorado School of Mines

Xu X and Tsvankin I 2006a Anisotropic geometrical-spreading correction for wide-azimuth P-wave reflections *Geophysics* **71** D161–70

Xu X and Tsvankin I 2006b Azimuthal AVO analysis with anisotropic spreading correction: a synthetic study *The Leading Edge* **25** 1336–42

Xu X and Tsvankin I 2007 A case study of azimuthal AVO analysis with anisotropic spreading correction *The Leading Edge* **26** 1552–61

Xu X, Tsvankin I and Pech A 2005 Geometrical spreading of P-waves in horizontally layered, azimuthally anisotropic media *Geophysics* **70** D43–53