

## Dip-moveout processing by Fourier transform in anisotropic media

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### ABSTRACT

Conventional dip-moveout (DMO) processing is designed for isotropic media and cannot handle angle-dependent velocity. We show that Hale's isotropic DMO algorithm remains valid for elliptical anisotropy but may lead to serious errors for nonelliptical models, even if velocity anisotropy is moderate.

Here, Hale's constant-velocity DMO method is extended to anisotropic media. The DMO operator, to be applied to common-offset data corrected for normal moveout (NMO), is based on the analytic expression for dip-dependent NMO velocity given by Tsvankin. Since DMO correction in anisotropic media requires knowledge of the velocity field, it should be preceded by an inversion procedure designed to obtain the normal-moveout velocity as a function of ray parameter. For transversely isotropic models with a vertical symmetry axis (VTI media), *P*-wave NMO velocity depends on a single anisotropic coefficient ( $\eta$ ) that can be determined from surface reflection data.

Impulse responses and synthetic examples for typical VTI media demonstrate the accuracy and efficiency of this DMO technique. Once the inversion step has been completed, the NMO-DMO sequence does not take any more computing time than the generic Hale method in isotropic media. Our DMO operator is not limited to vertical transverse isotropy as it can be applied in the same fashion in symmetry planes of more complicated anisotropic models such as orthorhombic.

### INTRODUCTION

Seismic anisotropy, widely recognized as a property of many subsurface formations of various origin (e.g., Thomsen, 1986), may have a strong influence on all seismic processing methods,

including dip-moveout algorithms (Levin, 1990; Larner, 1993; Tsvankin, 1995, 1996). Most constant-velocity dip-moveout (DMO) techniques are based on the cosine-of-dip dependence of moveout velocity on reflector dip, valid for homogeneous isotropic media (Levin, 1971):

$$V_{\text{nmo}}(\phi) = \frac{V_{\text{nmo}}(0)}{\cos \phi}, \quad (1)$$

where  $V_{\text{nmo}}$  is normal-moveout velocity calculated in the zero-spread limit, and  $\phi$  is the dip angle. In isotropic media, normal moveout (NMO) velocity from equation (1) accurately describes reflection traveltimes for any spreadlength because, in the absence of inhomogeneity, moveout is purely hyperbolic.

For the purpose of DMO processing, moveout velocity is usually expressed through the ray parameter (horizontal slowness)  $p(\phi)$  corresponding to the zero-offset reflection.

$$p(\phi) = \frac{1}{2} \frac{dt_0}{dy_0} = \frac{\sin \phi}{V}, \quad (2)$$

where  $t_0(y_0)$  is the two-way traveltime on the zero-offset (or stacked) section, and  $y_0$  is the midpoint position. Using equation (2) and taking into account that in isotropic media  $V_{\text{nmo}}(0) = V$ , equation (1) can be rewritten as

$$V_{\text{nmo}}(p) = \frac{V_{\text{nmo}}(0)}{\sqrt{1 - p^2 V_{\text{nmo}}^2(0)}}. \quad (3)$$

Angular velocity variations in anisotropic media lead to errors in the isotropic relationships (1) and (3). Tsvankin (1995) (hereafter referred to as Paper I) derived an analytic equation for NMO velocity from dipping reflectors in anisotropic media and studied the error of conventional DMO for transversely isotropic models. He concluded that the dip-dependence of the *P*-wave moveout velocity for transverse isotropy with a vertical symmetry axis (VTI media) is controlled, to a significant degree, by the difference between the Thomsen (1986) parameters  $\epsilon$  and  $\delta$ . If  $\epsilon - \delta > 0$  (typical case), the cosine-of-dip corrected moveout velocity may be larger significantly than the

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moveout velocity for a horizontal reflector. Even for moderate values of  $\epsilon - \delta = 0.1$  and  $\epsilon \leq 0.2$ , the error of the isotropic equation (1) reaches 25% at 45° dip and 30%–35% at a dip of 60°. The errors in equation (3) for positive  $\epsilon - \delta$  turn out to be even higher than those in equation (1) (Paper I).

Since dip-moveout removal is an important step in the conventional processing flow (NMO-DMO-poststack migration), any errors in DMO will propagate into the final seismic image (e.g., dipping reflectors may be missed or dimmed because of mistacking). Clearly, there is a need to develop dip-moveout algorithms capable of handling transversely isotropic media and, in the future, more complicated anisotropic models. Uren et al. (1990) showed that Forel and Gardner DMO (1988) can be updated and applied in a straightforward way to elliptically anisotropic models. However, elliptical anisotropy is no more than a special case of transverse isotropy that corresponds to  $\epsilon = \delta$  and cannot be considered as typical for real rocks (Thomsen, 1986). Also, Paper I demonstrates that the  $P$ -wave NMO velocity is highly sensitive to deviations from elliptical anisotropy.

The main difficulty in devising efficient DMO algorithms for anisotropic media was the absence of closed-form expressions for normal-moveout velocity necessary to replace the isotropic equations (1) and (3). This gap was filled by the NMO equation from Paper I, which can be used in the symmetry planes of any anisotropic medium. Although this equation was derived in the zero-spread limit, it provides an accurate description of  $P$ -wave moveout on conventional-length spreads close to the reflector depth. We have already used this analytic expression to devise a Fowler-type DMO algorithm for transversely isotropic media (Anderson et al., 1996).

Here, the equation for dip-dependent NMO velocity from Paper I is applied to extend Hale's (1984) DMO method to anisotropic media. We show that for vertical transverse isotropy the anisotropic correction to Hale's DMO operator is dependent mostly on the difference  $\epsilon - \delta$  and can make a significant contribution to DMO processing results for nonelliptical models. In addition to NMO correction, the anisotropic DMO removal in VTI media should be preceded by an inversion procedure intended to obtain the effective anisotropic coefficient ( $\eta$ ) responsible for the dependence of NMO velocity on the ray parameter. We study the complications caused by anisotropy in the DMO process and illustrate the performance of our algorithm by calculating impulse responses and processing synthetic sections for typical transversely isotropic models.

## ANALYTIC FORMULATION

### Hale's DMO method

First, we briefly review the fundamentals of Hale's (1984) frequency-wavenumber ( $F$ - $K$ ) DMO algorithm. For both isotropic and anisotropic media, the original event time  $t$  on conventional-length common-midpoint (CMP) spreads is close to a hyperbola parameterized by NMO velocity as

$$t^2 = t_0^2 + \frac{4h^2}{V_{\text{nmo}}^2(\phi)}, \quad (4)$$

where  $t_0$  is the two-way zero-offset traveltime, and  $h$  is half the source-receiver offset. The normal-moveout equation for

horizontal reflectors, applied to the data prior to DMO processing, results in the NMO-corrected time  $t_n$  given by

$$t_n^2 = t^2 - \frac{4h^2}{V_{\text{nmo}}^2(0)}. \quad (5)$$

Combining equations (4) and (5) leads to the following expression for the zero-offset time in terms of the NMO-corrected time:

$$t_0^2 = t_n^2 + 4h^2 \left[ \frac{1}{V_{\text{nmo}}^2(0)} - \frac{1}{V_{\text{nmo}}^2(\phi)} \right]. \quad (6)$$

This equation, valid both in isotropic media and for the hyperbolic portion of the moveout curve in anisotropic media, is required for the coordinate transformation used in Hale's DMO.

In a homogeneous, isotropic medium, equations (6) and (1) yield

$$t_0^2 = t_n^2 + \frac{4h^2 \sin^2 \phi}{V^2}, \quad (7)$$

or, introducing the ray parameter [equations (2) and (3)],

$$t_0^2 = t_n^2 + 4h^2 p^2. \quad (8)$$

Hale's isotropic DMO corrects for the remaining dip-dependent moveout term in equations (7) and (8) ( $4h^2 p^2$ ). Therefore, the process of generating the zero-offset section is split into the NMO and DMO steps, with the DMO operator being completely independent of velocity.

A convenient way to obtain the ray parameter is to transform the data into the frequency-wavenumber domain. Then, for the zero-offset wavefield we have

$$p = \frac{1}{2} \frac{dt_0}{dy_0} = \frac{k}{2\omega}, \quad (9)$$

where  $k$  is the horizontal spatial wavenumber over the common-midpoint axis, and  $\omega$  is the angular frequency associated with  $t_0$ .

Hale's (1984) DMO is based on the following Fourier-transform relationships between the time-space domain and the frequency-wavenumber domain for the zero-offset wavefield  $P_0$ :

$$P_0(t_0, y_0, h) = \int \frac{d\omega}{2\pi} \int \frac{dk}{2\pi} P_0(\omega, k, h) e^{-i(\omega t_0 - k y_0)}; \quad (10)$$

$$P_0(\omega, k, h) = \int dt_n \int dy_n P_n(t_n, y_n, h) J_T e^{i(\omega t_n A - k y_n)}, \quad (11)$$

where  $P_n(t_n, y_n, h)$  denotes the data after the normal-moveout correction,  $J_T$  is the transformation's Jacobian, and  $A = t_0/t_n$ . The relation between the zero-offset and common-offset NMO-corrected data is defined as  $P_0(t_0, y_0, h) \equiv P_n(t_n, y_n, h)$ .

The dip-dependence of normal-moveout velocity is contained in the term  $A$  that relates the zero-offset and NMO-corrected time at a given half-offset  $h$ . To emphasize the form that is also valid for anisotropic media, we use equation (6) to express  $A$  as a function of NMO velocity,

$$A = \sqrt{1 + \frac{4h^2}{t_n^2} \left[ \frac{1}{V_{\text{nmo}}^2(0)} - \frac{1}{V_{\text{nmo}}^2(p)} \right]}. \quad (12)$$

For isotropic media, equations (3) and (9) enable us to put  $A$  in the form used by Hale (1984), where

$$A = \sqrt{1 + \frac{4h^2 p^2}{t_n^2}} = \sqrt{1 + \frac{k^2 h^2}{\omega^2 t_n^2}}. \quad (13)$$

In Hale's (1984) original algorithm, it is assumed that reflection-point dispersal can be ignored, and  $y_0 \equiv y_n$ . A similar assumption is made in Paper I in the derivation of equation (17) for  $V_{\text{nmo}}(\phi)$  in anisotropic media. Therefore, the Jacobian  $J_T$  of the transformation (11) is defined as

$$J_T = \frac{\partial t_0}{\partial t_n} = \frac{t_n}{t_0} = A^{-1}. \quad (14)$$

Two "true-amplitude" variants of Hale's algorithm are of interest. Both variants correct Hale's identification  $y_0 \equiv y_n$ , make the corresponding modification of  $t_0$  [as defined in equation (4)], and thereby honor the movement of the surface location associated with a specular reflection point during the transformation to zero offset. The phase factors in equation (11) used by these "true-amplitude" algorithms coincide with Hale's phase factor, and each differs from the original Hale algorithm only in the value assumed for  $J_T$ . Both require a Jacobian based on  $\partial(t_0, y_0)/\partial(t_n, y_n)$  rather than Hale's more simple Jacobian,  $\partial t_0/\partial t_n$ . Bleistein (1990) assumes that (1) the input data have not been corrected for spherical divergence, (2) NMO is done by resampling without any scale factors, and (3) the spectral density of the input wavelet is preserved. The corresponding value for  $J_T$  in Bleistein's algorithm is

$$J_T = A^{-1}(2A^2 - 1). \quad (15)$$

Black et al. (1993) assume that (1) the input data have been corrected for spherical divergence, (2) NMO is done by resampling without any scale factors, and (3) the peak amplitude of the input wavelet is preserved. This leads to the following value for  $J_T$ :

$$J_T = A^{-3}(2A^2 - 1). \quad (16)$$

In essence, both "true-amplitude" algorithms correct a small amplitude error on the steeper dips associated with Hale's Jacobian for a chosen sequence of data processing steps. The kinematics of the DMO operator remain the same for all three Jacobians discussed above.

In the presence of anisotropy, the specular reflection point moves differently than in isotropic media, and we do not account for this movement in our algorithm. However, as shown in Paper I and further illustrated by synthetic examples below, for conventional spreads (close to the CMP-reflector distance or smaller) this simplification does not influence the kinematic aspects of the DMO correction.

### DMO for general anisotropy

Here, we modify Hale's (1984) DMO operator using the anisotropic NMO equation given in Paper I. The model considered in Paper I consists of a plane dipping reflector beneath a homogeneous anisotropic medium (Figure 1). Anisotropy is not restricted to any specific type; however, the incidence (sagittal) plane is supposed to be a plane of symmetry. For transversely isotropic media (the most common anisotropic model), the incidence plane should contain the symmetry axis.

If the medium is orthorhombic, the incidence plane is assumed to represent one of the mutually orthogonal symmetry planes.

If the symmetry assumption is satisfied, the dip-dependent normal-moveout velocity on a CMP line perpendicular to the strike of the reflector is given by (Paper I) as

$$V_{\text{nmo}}(\phi) = \frac{V(\phi)}{\cos \phi} \frac{\sqrt{1 + \frac{V''(\phi)}{V(\phi)}}}{1 - \tan \phi \frac{V'(\phi)}{V(\phi)}}, \quad (17)$$

where  $V$  is phase velocity as a function of phase angle with vertical. If the medium is isotropic, the derivatives of phase velocity vanish, and equation (17) reduces to the isotropic cosine-of-dip dependence (1).

Relation (2) between the dip  $\phi$  and ray parameter  $p$  on the zero-offset section continues to hold in anisotropic media (Paper I),

$$p(\phi) = \frac{1}{2} \frac{dt_0}{dy_0} = \frac{\sin \phi}{V(\phi)} = \frac{k}{2\omega}. \quad (18)$$

The replacement of the dip angle  $\phi$  by the ray parameter  $p$  for a known anisotropic model can be done in a straightforward fashion. The phase angle and phase velocity corresponding to a given value of  $p$  can be obtained from the Christoffel equation and used in formula (17) (Alkhalifah and Tsvankin, 1995).

While equation (17) is valid for all wave types (except for mode conversions), we will consider DMO processing for only quasi- $P$ -waves (in the following, we omit the qualifiers in "quasi- $P$ -wave" and "quasi- $S$ -wave" for brevity). To generalize Hale's DMO for anisotropic media, we need to separate the term associated with dipping reflectors in the anisotropic moveout equation. Although  $P$ -wave reflection moveout even in homogeneous anisotropic media is generally nonhyperbolic (Tsvankin and Thomsen, 1994), it can still be approximated by a hyperbola on spreads with length comparable to the distance

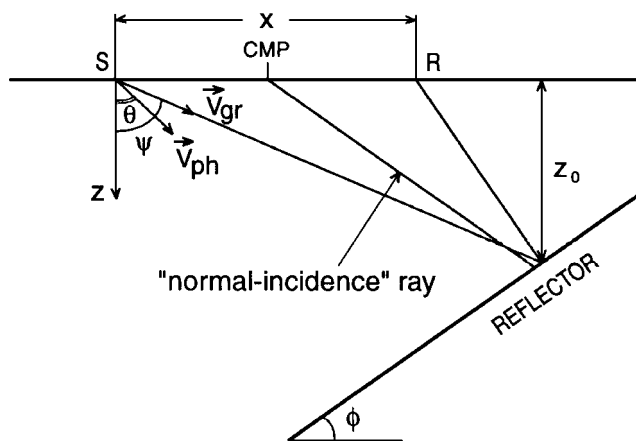


FIG. 1. Common-midpoint gather over a homogeneous anisotropic medium (after Paper I).  $\mathbf{V}_{\text{gr}}$  and  $\mathbf{V}_{\text{ph}}$  are the group- and phase-velocity vectors, respectively. The phase-velocity vector corresponding to the zero-offset ("normal-incidence") ray is perpendicular to the reflector.

between the CMP and the reflector (Paper I). Hence, we can still use the conventional hyperbolic equation (4)

$$t^2 = t_0^2 + \frac{4h^2}{V_{\text{nmo}}^2(p)}, \quad (19)$$

but with the NMO velocity given by equation (17). Expression (17) is too complicated to be separated analytically into the zero-dip velocity and the DMO correction factor, but we can accomplish this separation artificially by rewriting equation (19) in the same form as equation (6).

$$t_0^2 = t_n^2 + 4h^2 \left[ \frac{1}{V_{\text{nmo}}^2(0)} - \frac{1}{V_{\text{nmo}}^2(p)} \right]. \quad (20)$$

If the medium were isotropic and homogeneous, equation (20) would reduce to the conventional DMO formula (8).

Therefore, the zero-offset section in anisotropic media can be produced by an NMO-DMO sequence similar to the one used in conventional processing. The time  $t_n$  is a result of the conventional NMO procedure that eliminates the zero-dip (NMO) term  $4h^2/V_{\text{nmo}}^2(0)$ . The influence of anisotropy in the normal-moveout correction is hidden in the value of  $V_{\text{nmo}}(0)$ . The only possible complication at this stage is anisotropy-induced deviations from hyperbolic moveout, which are small on conventional-length CMP spreads.

To transform the NMO-corrected data to zero offset, the anisotropic DMO operator should compensate for the remaining moveout term  $4h^2 D(p)$  in equation (20), with the factor  $D(p)$  given by

$$D(p) = \frac{1}{V_{\text{nmo}}^2(0)} - \frac{1}{V_{\text{nmo}}^2(p)}. \quad (21)$$

The main difference between the anisotropic DMO factor  $D(p)$  and its isotropic counterpart  $p^2$  [equation (8)] is that  $D(p)$  has to be obtained from equation (17) that depends on the parameters of the anisotropic velocity field. Therefore, DMO processing should be preceded by an inversion procedure designed to reconstruct the NMO velocity as a function of ray parameter. For transverse isotropy with a vertical symmetry axis (VTI),  $P$ -wave surface data alone provide enough information to build the dip-dependent NMO velocity (Alkhalifah and Tsvankin, 1995). The method developed by Alkhalifah and Tsvankin (1995) is used in our implementation of Hale's DMO in VTI media.

Assuming that the functions  $V_{\text{nmo}}(p)$  and  $D(p)$  have been obtained, the rest of Hale's DMO algorithm remains essentially unchanged. The DMO factor  $D(p)$  should be substituted into the equation for  $A$  (12), yielding the expression valid for anisotropic media:

$$A = \sqrt{1 + \frac{4h^2}{t_n^2} D(p)}. \quad (22)$$

Equation (22) is then used in the integral transforms (11) and (10) designed to produce the zero-offset section from the NMO-corrected, constant-offset sections. This means that the entire anisotropic NMO-DMO sequence takes no more computing time than does Hale's method for isotropic media. The only extra step to be included because of the presence of anisotropy is the inversion procedure mentioned above.

Our algorithm ignores anisotropy-induced nonhyperbolic moveout, which seems to be a shortcoming compared to

isotropic DMO methods. Although isotropic constant-velocity DMO is also based on the hyperbolic moveout equation, it is supposed to work at large offsets because reflection moveout in a homogeneous isotropic medium is purely hyperbolic. However, the homogeneous isotropic model is an idealization of realistic media, which are, at a minimum, vertically inhomogeneous. Any kind of inhomogeneity leads to nonhyperbolic moveout on long spreads that cannot be handled properly by conventional NMO and DMO algorithms.

Also, the results of Paper I and numerical examples for VTI media discussed below suggest that for  $P$ -waves deviations from hyperbolic moveout on conventional-length spreads are relatively small and usually become even less pronounced with dip. Therefore, nonhyperbolic moveout can be expected to be more of a problem in the NMO correction for horizontal reflectors than in dip-moveout removal. Relatively large offsets, at which deviations from a hyperbola may become significant, are often muted out because of NMO stretch. If there is a need to preserve traces at these uncommonly large offsets, DMO processing should be performed by more accurate (but more costly) ray-tracing techniques.

### Transversely isotropic media

While NMO equation (17) can be used in symmetry planes of any anisotropic medium, the main practical difficulty in the implementation of our DMO algorithm in complicated anisotropic models is the recovery of the factor  $D(p)$  from seismic data. Therefore, in the following we concentrate on the most common anisotropic model, transverse isotropy with a vertical symmetry axis (VTI).

We describe VTI media by four Thomsen (1986) parameters: the vertical velocities  $V_{P0}$  and  $V_{S0}$  of  $P$ - and  $S$ -waves, respectively, and three dimensionless anisotropic coefficients— $\epsilon$ ,  $\delta$ , and  $\gamma$ . It should be emphasized that although the original Thomsen (1986) paper was devoted to weak transverse isotropy, the parameters  $\epsilon$ ,  $\delta$ , and  $\gamma$  are convenient to use in TI media with arbitrary strength of velocity anisotropy (Tsvankin, 1996). In our DMO algorithm, we use the exact equations for transverse isotropy without applying the weak-anisotropy approximation.

Although  $P$ - $SV$  propagation depends on four Thomsen coefficients ( $V_{P0}$ ,  $V_{S0}$ ,  $\epsilon$ , and  $\delta$ ),  $P$ -wave velocities and reflection traveltimes are practically independent of the shear-wave vertical velocity  $V_{S0}$ , even for strong anisotropy (Tsvankin and Thomsen, 1994; Tsvankin, 1995, 1996). Therefore, for practical purposes of dip-moveout processing,  $P$ -wave NMO velocity can be considered a function of just three parameters:  $V_{P0}$ ,  $\epsilon$ , and  $\delta$  (Paper I).

Moreover, Alkhalifah and Tsvankin (1995) show that  $P$ -wave NMO velocity in VTI media (expressed as a function of the ray parameter) depends on only *two* combinations of these three coefficients—the zero-dip NMO velocity  $V_{\text{nmo}}(0)$  and an anisotropic parameter, denoted as  $\eta$ :

$$V_{\text{nmo}}(0) = V_{P0} \sqrt{1 + 2\delta}, \quad (23)$$

$$\eta \equiv \frac{\epsilon - \delta}{1 + 2\delta} = \frac{1}{2} \left[ \frac{V_P^2(90)}{V_{\text{nmo}}^2(0)} - 1 \right], \quad (24)$$

where  $V_P(90)$  is the horizontal  $P$ -wave velocity given by

$$V_P(90) = V_{P0}\sqrt{1+2\epsilon} = V_{\text{nmo}}(0)\sqrt{1+2\eta}. \quad (25)$$

The velocity-analysis technique by Alkhalifah and Tsvankin (1995) is designed to recover the parameters  $V_{\text{nmo}}(0)$  and  $\eta$  using  $P$ -wave NMO velocities from two different dips. If one of the reflectors is horizontal (the most common case), then  $V_{\text{nmo}}(0)$  is obtained directly by conventional velocity (e.g., semblance) analysis. The NMO velocity for the dipping reflector is then used to determine  $\eta$  and build the normal-moveout velocity as a function of the ray parameter.

To understand the performance of the anisotropic DMO algorithm, we have to examine the dependence of the factor  $D(p)$  [equation (21)] on the parameters of the anisotropic velocity field. First, we consider elliptically anisotropic models, in which wavefronts and slowness surfaces are elliptical for  $P$ -waves and spherical for  $SV$ -waves. Elliptical anisotropy is a special (and uncommon) case of transverse isotropy that occurs if the parameters  $\epsilon$  and  $\delta$  are equal to each other (and  $\eta = 0$ ).

Alkhalifah and Tsvankin (1995) show that for elliptical anisotropy  $P$ -wave NMO velocity represents the same function of the ray parameter and zero-dip moveout velocity as in isotropic media (3), with no explicit dependence on the coefficient  $\epsilon = \delta$ . The difference between the isotropic and elliptically anisotropic expressions for  $V_{\text{nmo}}(p)$  is only in the value of the zero-dip velocity  $V_{\text{nmo}}(0)$  [equation (23)], which is equal to the horizontal velocity (but is different from the vertical velocity) if the medium is elliptically anisotropic. Since the isotropic expression for  $V_{\text{nmo}}(p)$  holds in elliptical media, conventional constant-velocity Hale's DMO is perfectly suitable for elliptical anisotropy. This result is not intuitively clear because the NMO velocity as a function of dip in elliptically anisotropic media does deviate from the cosine-of-dip dependence (Uren et al, 1990; Paper I).

Furthermore, the entire isotropic time-imaging sequence (NMO-DMO-time migration) remains valid for elliptical anisotropy (Helbig, 1983; Alkhalifah and Tsvankin, 1995). The difference between isotropic and elliptically anisotropic media becomes important only in time-to-depth conversion or post-stack depth migration of the zero-offset section. Since for elliptical anisotropy the zero-dip NMO velocity, conventionally used in time-to-depth conversion or migration, does not coincide with the true vertical velocity, isotropic poststack depth migration will result in the wrong reflector depth.

While the elliptical assumption leads to a considerable simplification in the analysis of all aspects of wave propagation, elliptical anisotropy is hardly typical for subsurface formations (Thomsen, 1986). Next, we consider a general transversely isotropic model with no fixed relation between  $\epsilon$  and  $\delta$ .

The most convenient way to get analytic insight into the dependence of  $D(p)$  on the anisotropic coefficients is to use the weak-anisotropy approximation ( $|\epsilon| \ll 1$ ,  $|\delta| \ll 1$ ). The  $P$ -wave phase velocity, linearized in the parameters  $\epsilon$  and  $\delta$ , is given by (Thomsen, 1986)

$$V_P(\theta) = V_{P0}(1 + \delta \sin^2 \theta \cos^2 \theta + \epsilon \sin^4 \theta). \quad (26)$$

Substitution of equation (26) into the NMO formula (17) and replacement of the phase angle by ray parameter yields (Alkhalifah and Tsvankin, 1995)

$$V_{\text{nmo}}(p) = \frac{V_{\text{nmo}}(0)}{\sqrt{1 - p^2 V_{\text{nmo}}^2(0)}} [1 + (\epsilon - \delta)f(pV_{\text{nmo}}(0))], \quad (27)$$

$$f \equiv \frac{y(4y^2 - 9y + 6)}{1 - y}; \quad y \equiv p^2 V_{\text{nmo}}^2(0).$$

In the derivation of equation (27) it was assumed that  $y < 1$  (e.g., the dip angle cannot be close to  $90^\circ$ ). The weak-anisotropy approximation (27) reduces to the exact NMO formula for elliptically anisotropic or isotropic models [ $\epsilon = \delta$ , equation (3)] but, clearly, can deviate from the exact result for nonelliptical media.

The DMO correction factor  $D(p)$  (21) for weak transverse isotropy becomes

$$D(p) = p^2 [1 + 2(\epsilon - \delta)(4y^2 - 9y + 6)]. \quad (28)$$

Note that the anisotropic correction to the DMO operator in the weak-anisotropy approximation contains only the difference  $\epsilon - \delta$ , with no separate dependence on either of the coefficients. The predominant influence of  $\epsilon - \delta$  on the dip-dependence of  $P$ -wave NMO velocity was shown in Paper I. It should be mentioned, however, that the weak-anisotropy NMO formula derived in Paper I as a function of the dip angle did contain a separate contribution of the parameter  $\delta$ . As demonstrated by equations (27) and (28), this contribution is absorbed by the ray parameter  $p$ .

The numerical analysis by Alkhalifah and Tsvankin (1995) shows that the structure of the factor  $D(p)$  (21) for VTI media with arbitrary strength of velocity anisotropy is similar to that in equation (28), with the influence of the anisotropic coefficients  $\epsilon$  and  $\delta$  represented by the effective parameter  $\eta$ . Clearly,  $\eta$  [equation (24)] reduces to  $\epsilon - \delta$  in the limit of weak anisotropy.

Although it might be dangerous to use the weak-anisotropy approximation for quantitative estimates, it is clear from equation (28) that the anisotropy makes a significant contribution to the DMO factor. For instance, for  $y = 0.25$ , which roughly corresponds to a dip of  $30^\circ$ , the anisotropic term  $2(\epsilon - \delta)(4y^2 - 9y + 6)$  reduces to  $8(\epsilon - \delta)$ , which means that the isotropic and anisotropic parts of the DMO factor are comparable even for relatively small  $(\epsilon - \delta)$ . This conclusion is in good agreement with the exact numerical results of Paper I.

It is also important that the influence of the anisotropy cannot be ignored even at mild dips (small  $y$ ); in fact, the relative magnitude of the anisotropic term *decreases* with dip. Of course, the contribution of the dip term as a whole at mild dips is relatively small. Our analysis also shows that, although the anisotropic DMO operator does depend on the zero-dip NMO velocity through  $y \equiv p^2 V_{\text{nmo}}^2(0)$ , the influence of errors in  $V_{\text{nmo}}(0)$  becomes pronounced only for large  $p$  corresponding to steep events.

It should be mentioned that most measurements made at seismic frequencies indicate that  $\epsilon > \delta$  (Thomsen, 1986; Sayers, 1994) and, therefore,  $\eta > 0$ . For positive  $\eta$  [and a positive anisotropic term in equation (28)], normal-moveout velocity increases with  $p$  faster than in isotropic media (Paper I). As a result, conventional isotropic DMO algorithms usually underestimate NMO velocity for dipping events in VTI media, which was illustrated on synthetic and field data in Alkhalifah and Tsvankin (1995).

### NUMERICAL IMPLEMENTATION

The DMO technique for VTI media discussed above has been implemented using a variant of the conventional Hale  $F$ - $K$  DMO algorithm. First,  $V_{\text{nmo}}(\phi)$  and  $p = \sin \phi / V(\phi)$  are tabulated from 0 to 89° in constant increments of  $\phi$  using equation (17). The input parameters to be determined beforehand are the NMO velocity for a horizontal reflector  $V_{\text{nmo}}(0)$  and the coefficient  $\eta$  [equation (24)]. We use the exact equations for NMO and phase velocity in VTI media, so there are no restrictions requiring weak anisotropy in the generation of this table. The table is then interpolated back to a new table in constant increments in  $p$ . Alternatively, the function  $V_{\text{nmo}}(p)$  can be built directly using phase-velocity equations for transverse isotropy. Based upon this table,  $V_{\text{nmo}}(p)$  and  $D(p)$  can be readily obtained.

Input NMO-corrected common-offset data are Fourier-transformed over the midpoint axis providing complex-valued traces as a function of wavenumber. The time integral over  $t_n$  is computed numerically, yielding the zero-offset section in the  $F$ - $K$  ( $\omega - k$ ) domain [equation (11)].

$$P_0(\omega, k, h) = \int dt_n \frac{P_n(t_n, k, h)}{A} e^{i\omega t_n A}, \quad (29)$$

where  $p = k/2\omega$ , and [see equation (22)]

$$\begin{aligned} A(t_n, k, \omega) &= \sqrt{1 + \frac{4h^2}{t_n^2} D(p)} \\ &= \sqrt{1 + \frac{4h^2}{t_n^2} \left[ \frac{1}{V_{\text{nmo}}^2(0)} - \frac{1}{V_{\text{nmo}}^2(p)} \right]}. \end{aligned}$$

Finally, we apply a 2-D inverse Fourier transform to produce the output DMO-corrected, common-offset gather in the  $t - x$  domain.

The direct numerical integration over  $t_n$  [equation (29)], currently implemented in our code, is the most time-consuming part of the algorithm. In the future, we plan to study the possibility of reducing the integral (29) to a Fourier transform by the log-stretch method (Notfors and Godfrey, 1987; Liner, 1990), which is used widely to speed up application of Hale DMO in isotropic media. The log-stretch temporal relationships, which are exact for homogeneous isotropic media, become only approximate in the presence of transverse isotropy.

### SYNTHETIC EXAMPLES

Figures 2-7 demonstrate basic aspects of our Hale-type DMO operator devised for vertical transverse isotropy. As expected, the VTI DMO algorithm produces results identical to conventional Hale DMO if the medium is isotropic (Figure 2a,b). As discussed above, our DMO code is based currently on the original Hale's Jacobian. Bleistein's "true-amplitude" impulse response (Figure 2c) has the same shape as Hale's response but shows slightly higher amplitudes on the steeper dips.

In agreement with the analytic results discussed above, the VTI operator reduces to the conventional Hale's operator not only for isotropic but also for elliptically anisotropic media ( $\epsilon = \delta$ , Figure 3). Although velocity anisotropy for the model in Figure 3 is quite pronounced ( $\epsilon = \delta = 0.2$ ), it has no influence on the kinematics of the DMO process. The only difference

between the two DMO operators is that in the Hale algorithm the NMO velocity is calculated analytically while the VTI DMO response is generated using the values of  $V_{\text{nmo}}$  from table lookup.

However, even relatively small deviations from elliptical anisotropy have strong influence on the DMO operator. As mentioned above, it is believed that transversely isotropic subsurface formations, such as shales, typically have positive values of  $\epsilon - \delta$ . Figure 4 demonstrates the broadening of the DMO ellipse that occurs for a transversely isotropic model with a large positive  $\epsilon - \delta$ . Evidently, the DMO impulse response

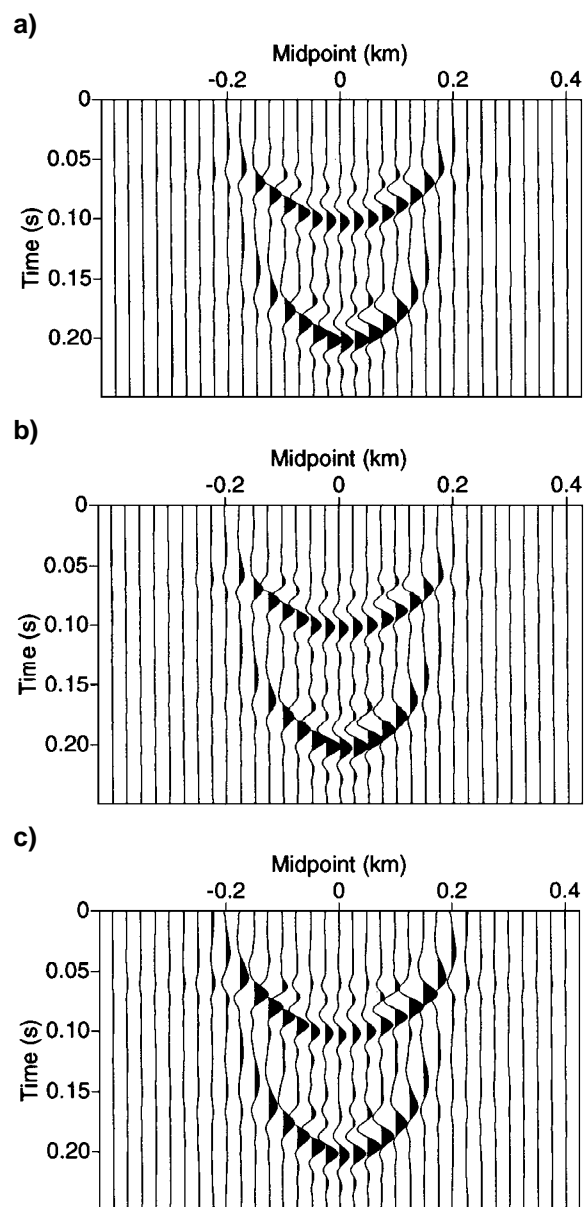


FIG. 2. DMO impulse responses on a common-offset gather for a homogeneous isotropic medium. Here, and in Figures 3-7, the offset equals 400 m and the trace spacing is 25 m. (a) Conventional Hale operator; (b) Tsvankin (VTI) operator (the VTI DMO response is identical to the Hale response for isotropic models); (c) Bleistein operator (the Bleistein "true-amplitude" DMO response has the same kinematics but stronger amplitudes at larger dips).

in anisotropic media is not limited necessarily to the source-receiver interval.

For small negative values of  $\epsilon - \delta$ , the DMO response becomes more narrow as shown in Figure 5. With increasing absolute value of  $\epsilon - \delta$  (for  $\epsilon - \delta < 0$ ), the DMO response curves

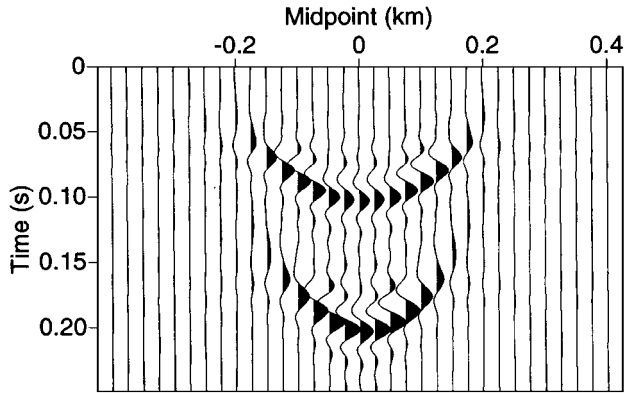


FIG. 3. VTI DMO impulse-response curves on a commonoffset gather for an elliptically anisotropic medium with  $\delta = \epsilon = 0.2$  ( $\eta = 0$ ). The DMO operator for elliptical anisotropy is kinematically identical to the isotropic DMO operator for any strength of the anisotropy.

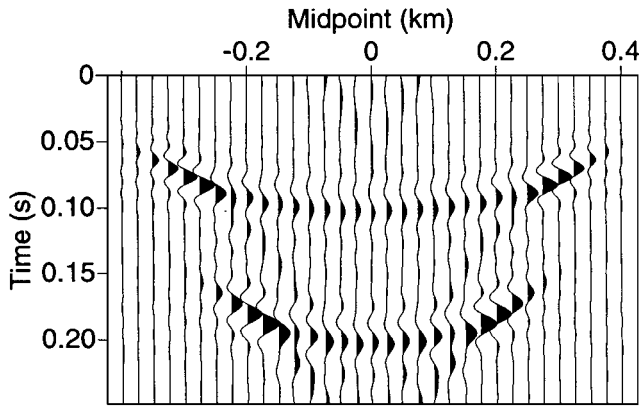


FIG. 4. VTI DMO impulse responses on a common-offset gather for  $\epsilon = 0.4$  and  $\delta = 0.0$  ( $\eta = 0.4$ ). The anisotropic DMO operator for positive values of  $\eta$  is more broad than the isotropic one and may have triplications.

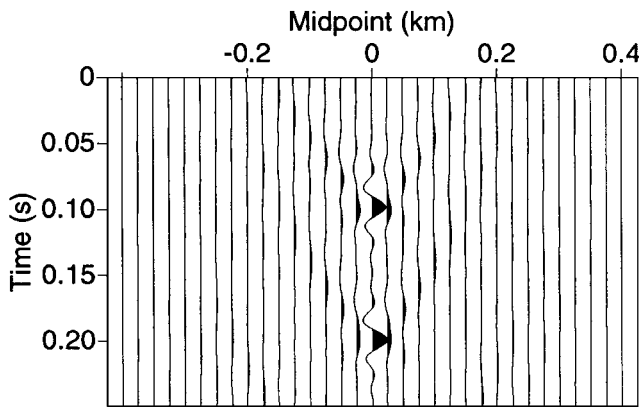


FIG. 5. VTI DMO impulse responses on a common-offset gather for  $\epsilon = 0.0$  and  $\delta = 0.1$  ( $\eta = -0.08$ ). The operator becomes more narrow for small negative values of  $\eta$ .

downward with a shape reminiscent of the isotropic inverse DMO operator (Figure 6).

The accuracy of the weak-anisotropy approximation (28) in generating the DMO operator is illustrated by Figure 7. For this model with moderate values of  $\epsilon$  and  $\delta$ , the DMO responses are similar, whether or not the weak-anisotropy equation is used. However, the weak-anisotropy approximation becomes less accurate for larger values of  $\epsilon - \delta$ , and we use the exact VTI operator in our routine numerical work.

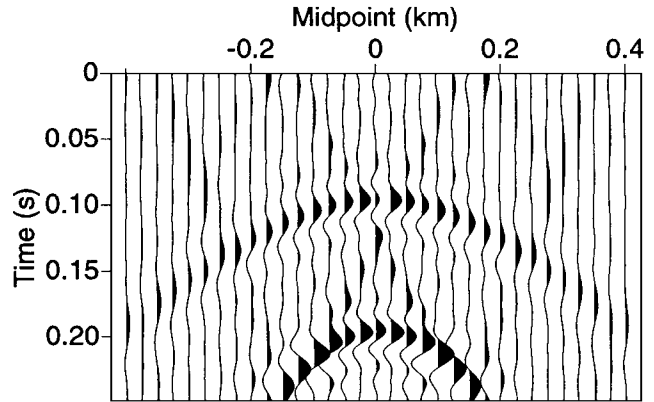


FIG. 6. VTI DMO impulse responses on a common-offset gather for  $\epsilon = 0.0$  and  $\delta = 0.2$  ( $\eta = -0.14$ ). For larger negative values of  $\eta$ , the operator can curve downward rather than upward.

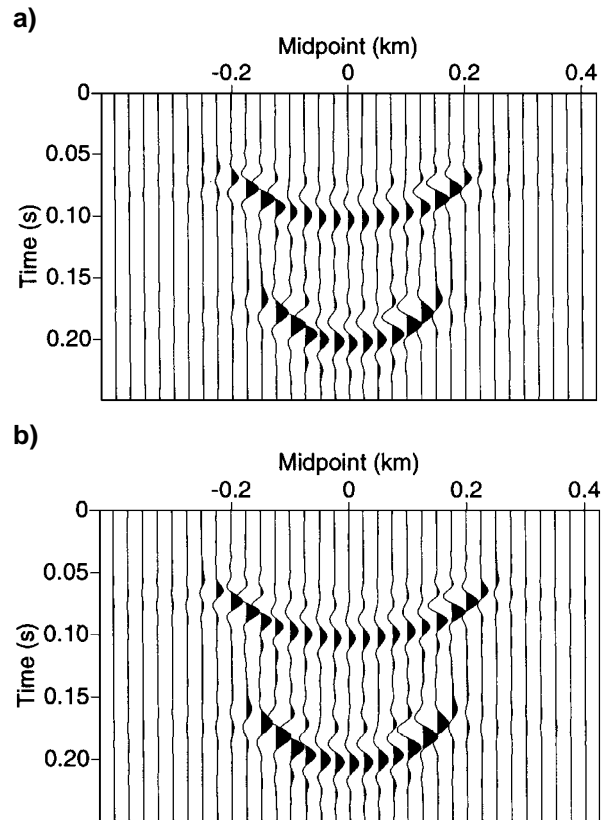


FIG. 7. VTI DMO impulse responses on a common-offset gather for  $\epsilon = 0.2$  and  $\delta = 0.1$  ( $\eta = 0.083$ ). (a) the response calculated using the exact NMO equation (17); (b) the response based on the weak-anisotropy approximation (28).

The next example shows the performance of the VTI DMO algorithm in a homogeneous VTI “fan” model with four reflectors and dips ranging from 0 to 75° (Figure 8). The Alkhalifah and Tsvankin model parameters are  $V_{nmo}(0) = 3.15$  km/s and  $\eta = 0.14$ ; the Thomsen parameters are  $V_{p0} = 3$  km/s,  $\epsilon = 0.2$ , and  $\delta = 0.05$ . Synthetic common-offset data for offsets from 0 to 3 km in increments of 0.1 km were generated by a ray-tracing code (Alkhalifah, 1995). The zero-offset section that may be considered as the ideal output for NMO followed by DMO is shown in Figure 9. The vertical line at midpoint location 2.75 km marks the CMP location selected for the comparison of isotropic and VTI DMO. Figure 9 helps to identify the key model events on the CMP gathers with the same events on the zero-offset section.

The NMO-corrected common-offset data were processed first by the conventional isotropic Hale DMO method (Figure 10), and then by our new VTI DMO algorithm (Figure 11). Evidently, VTI DMO aligns the events out to a fairly large offset, irrespective of dip, allowing CMP stack to sum without distortion. In contrast, after application of isotropic DMO in Figure 10, all dipping events remain misaligned. It should be emphasized that the synthetic data were generated by a ray-tracing algorithm that is not related to the analytic formula used in our anisotropic DMO correction.

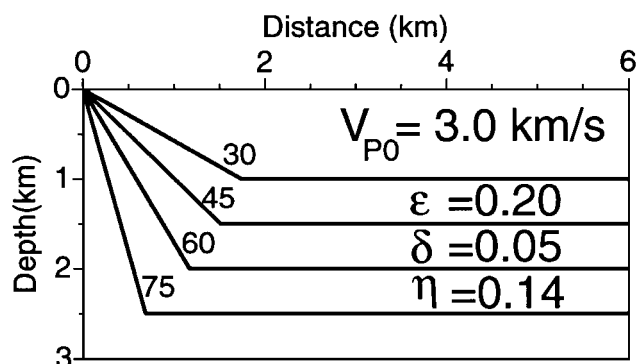


FIG. 8. The model used for generating synthetic data. The reflectors are embedded in a homogeneous transversely isotropic medium with a vertical symmetry axis.

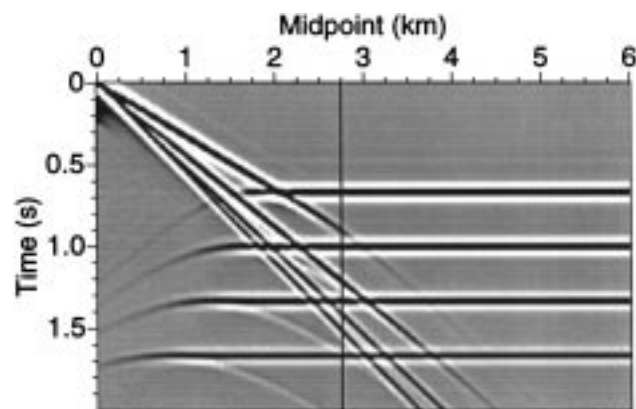


FIG. 9. Zero-offset synthetic data generated by ray tracing. The vertical line at 2.75 km marks the common-midpoint location used for comparing isotropic and VTI DMO corrections in Figures 10 and 11.

Figure 11 shows that the horizontal events still have significant nonhyperbolic moveout and will require muting on the longer offsets prior to the stack. It should be noted, however, that this example exaggerates the influence of nonhyperbolic

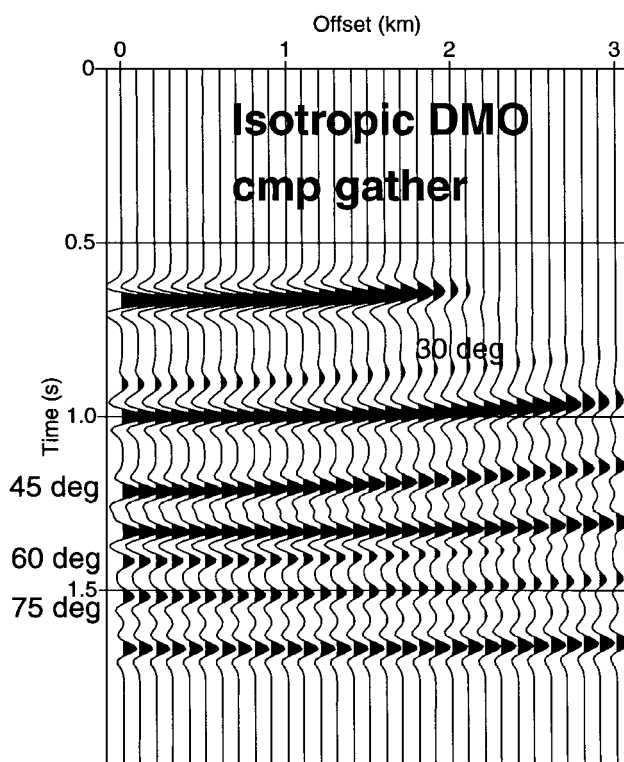


FIG. 10. CMP gather after NMO and isotropic DMO.

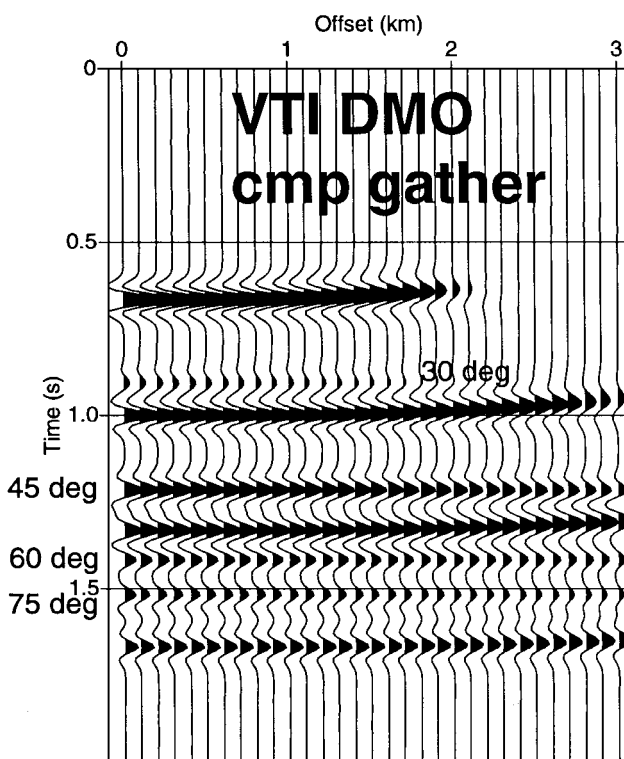


FIG. 11. CMP gather after NMO and VTI DMO.



moveout because standard processing rarely includes offsets out to 3 km at times less than 1.5 s. The dipping events also exhibit nonhyperbolic moveout, but to a far lesser degree than the horizontal events. It is clear from Figure 11 that our DMO operator, based on the hyperbolic moveout equation, still provides a sufficiently accurate description of moveout from dipping reflectors on relatively long spreads (twice as large as the reflector depth). If it is necessary to include even larger offsets in the stack, an NMO/DMO sequence based on anisotropic ray tracing may be necessary to correct for nonhyperbolic moveout.

#### POSSIBLE EXTENSIONS OF THE ALGORITHM

Application of the table lookup makes our algorithm no slower than the generic Hale DMO method in isotropic media. However, direct numerical integration over the NMO-corrected time employed in our code makes the DMO operation relatively time-consuming. The conventional way to speed up Hale's isotropic DMO algorithm is to use the log-stretch method (Notfors and Godfrey, 1987; Liner, 1990), but generalization of this method to anisotropic media is not straightforward as only the kinematics of the small-offset near-elliptical portion of the DMO impulse response can be preserved. The variations of amplitude and triplications seen along the anisotropic DMO response cannot be duplicated fully by straightforward stretching and squeezing in the log-stretch domain. In the future, we will explore the possibility of applying the log-stretch method in VTI media to increase the computational efficiency of this new DMO technique while simultaneously attempting to estimate the magnitude of the associated errors.

The algorithm can be upgraded to perform  $V(z)$  DMO in transversely isotropic media by replacing the single-layer NMO formula with a more general equation for layered media developed by Alkhalifah and Tsvankin (1995). However, since our "homogeneous" VTI DMO operator has the degree of freedom provided by the parameter  $\eta$ , in many cases it can be adapted for  $V(z)$  media by applying an effective value of  $\eta$  that absorbs the influence of both the anisotropy and inhomogeneity.

Also, our anisotropic DMO method can be used in symmetry planes of more complicated models such as orthorhombic. The main problem in DMO processing for anisotropic models with a large number of independent parameters is the reconstruction of the dip-dependence of NMO velocity from seismic data.

#### CONCLUSIONS

Hale's DMO method has been extended to anisotropic media using the analytic expression for normal-moveout velocity given in Tsvankin (1995). The NMO-DMO processing sequence (including Hale's integral transforms) remains unchanged in the presence of anisotropy, but the transformation's Jacobian has been modified to account for the anisotropic behavior of the dip dependence of NMO velocity.

For elliptically anisotropic media the isotropic DMO method remains entirely valid, although isotropic poststack depth migration based on the zero-dip moveout velocity may result in depth errors. However, elliptical anisotropy is just a special kind of transverse isotropy that cannot be considered as typical

for subsurface formations, and even relatively small deviations from the elliptical model may result in significant DMO errors.

We have implemented the  $F$ - $K$  DMO algorithm for the most common anisotropic model—transverse isotropy with a vertical symmetry axis (VTI media). The anisotropic correction to the DMO operator for VTI media is determined by two effective parameters: the NMO velocity for reflections from a horizontal interface (obtained by conventional velocity analysis) and the anisotropic coefficient  $\eta$  that can be recovered from the  $P$ -wave NMO velocity for a single dipping reflector (Alkhalifah and Tsvankin, 1995). After the inversion step has been completed, the analytic NMO equation is used to tabulate the normal-moveout velocity as a function of the ray parameter and obtain Hale's Jacobian for VTI media. Then the transformation of NMO-corrected constant-offset sections to zero offset is carried out using the formalism developed by Hale for isotropic media. Except for the inversion procedure, the entire anisotropic NMO-DMO sequence takes no more computing time than the conventional NMO and DMO corrections. Note that knowledge of  $\eta$  and  $V_{\text{nmo}}(0)$  is sufficient not only for DMO processing, but also for poststack time migration of the zero-offset section.

Tests on synthetic data for typical VTI media show that our VTI DMO operator efficiently flattens  $P$ -wave moveout on conventional-length spreads for any dip, while the isotropic Hale's algorithm fails to align dipping events in the presence of nonelliptical anisotropy. Two interesting points are clarified by the examples. First, reflections from steep interfaces are less influenced by nonhyperbolic moveout and appear more flat after DMO than are those from horizontal reflectors. This conclusion is in good agreement with the results reported in Tsvankin (1995). Therefore, we expect nonhyperbolic moveout (ignored in our algorithm) to be more of a problem in normal-moveout correction than in DMO. Second, the DMO response in strongly anisotropic media is not restricted to the region between the source and the receiver.

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