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Frank Rizzo and boundary integral equations

G. Fairweather [a], P.A. Martin [b,*], T.J. Rudolphi [c]

[a] Mathematical Reviews, American Mathematical Society, Ann Arbor, MI 48103, USA
[b] Department of Applied Mathematics and Statistics, Colorado School of Mines, Golden, CO 80401, USA
[c] Department of Aerospace Engineering, Iowa State University, Ames, IA 50011, USA

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ABSTRACT

Frank Rizzo was a pioneer in what he called boundary integral equation methods. With his students and colleagues, he developed theory and algorithms for many problems of engineering interest. In this memorial paper, we review his life and work. A list of his publications is included.

1. Introduction

Frank Joseph Rizzo was born in Chicago, Illinois, on April 20, 1938. [He was the only child of Frank J. Rizzo Sr. and Marie Rizzo, both offspring of Italian immigrants.] He attended schools in Chicago, Illinois, where he was graduated from St. Rita High School in 1955. After attending the Chicago Branch of the University of Illinois for two years, he enrolled at the Urbana Campus of the University of Illinois from which received his Bachelor of Science degree in February, 1960, and his Master of Science degree in August, 1961. While persuing the Master's degree and Doctor of Philosophy degree he was employed as a half time member of the teaching staff in the Department of Theoretical and Applied Mechanics.

This is taken from the last (the 30th) page of Frank's PhD thesis [R1],[1] dated October 1964. His advisor was Marvin Stippes (1922–1979), founding editor of the *Journal of Elasticity*. "Marvin was very fond of classical work...[and he]...wanted his students to be acquainted with the fundamental contributions of the early Italian elasticians such as Betti, Lauricella, and Somigliana...In spring 1963 Marvin suggested that 'something with integral equations' might be fruitful for me. He pointed me in the direction of Lovitt, Fredholm, a recent paper by Jaswon [3] and promptly left to go on a sabbatical for a year" [R43].

Frank's 1964 thesis led to his first paper [R2] published in early 1967. (Much of his subsequent work can be traced back to these publications; we will discuss their contents in Section 2.) During this period, Frank moved to the University of Washington (UW) with his wife, Mary Lou (Fig. 1), and their two young sons, Robert and Russell, where he was an assistant professor of civil engineering for two years. It was at

Fig. 1. Frank and Mary Lou Rizzo.

UW that he met Tom Cruse. He "showed in his thesis [1] that boundary integral equations, with element-methods of solution, were an effective vehicle for numerically treating time-dependent elasticity problems. Until then in elasticity, static problems in 2-D only had been attempted by others with such methods" [R95]. Cruse has explained how he was taking "an elastic wave propagation course taught by Professor Rizzo....One day, Frank showed [the class] how the Laplace transform converted the

* Corresponding author.
E-mail addresses: pamartin@mines.edu (P.A. Martin), rudolphi@iastate.edu (T.J. Rudolphi).

[1] A citation of the form [Rxx] indicates one of Rizzo's publications, listed in Section 7. Citations without the prefix R are to the list of references at the end of the paper.

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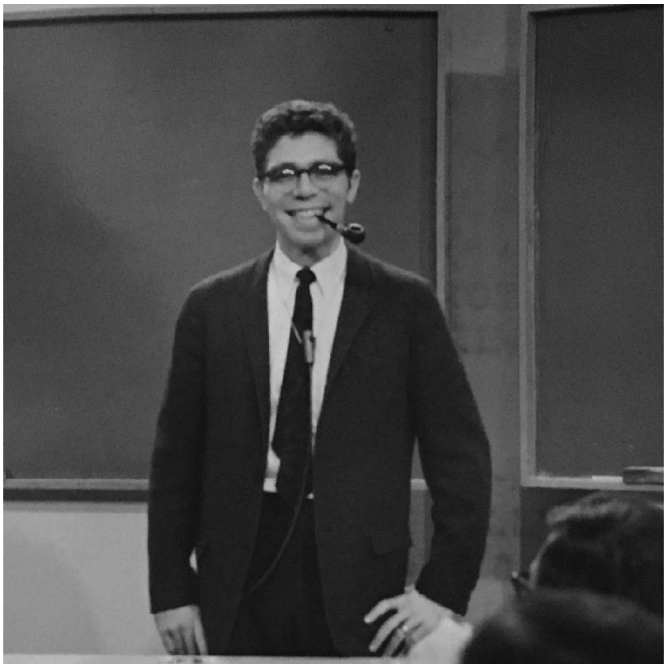

Fig. 2. Frank preparing to teach a class.

hyperbolic wave equation into an elliptic equation – I found my research topic at that point" [2].

By the fall of 1966, Frank and his family had moved from Seattle to Lexington where they were soon joined by a baby daughter, Kristen. Frank began as an assistant professor in the Department of Engineering Mechanics at the University of Kentucky (UK). He stayed at UK for 21 years, collaborating productively with a colleague, David Shippy (1931–2010). They "made steady if not overly rapid progress with two-dimensional applications in heat conduction [R6], anisotropic elasticity [R5], isotropic inclusion problems [R4], and viscoelasticity [R7]" [R43]. Shippy described their collaboration in some detail: "Because I enjoyed the computer work much more than Frank did, I did most of the actual coding" [7]. We shall return to the Kentucky years in Section 3.

In the summer of 1987, Frank and Mary Lou moved to Ames, Iowa, where he was appointed as Professor and Chairman of the Department of Engineering Science and Mechanics at Iowa State University (ISU); see Fig. 3. This move more-or-less coincides with his interest in crack problems and their treatment using hypersingular integral equations (Section 4). After two years, he was persuaded to return to his alma mater: he became Professor and Head of the Department of Theoretical and Applied Mechanics at the University of Illinois (UIUC). However, by 1992, he was back at ISU, as Professor in the Department of Aerospace Engineering and Engineering Mechanics, and as a Member of the Center for Nondestructive Evaluation (CNDE). The CNDE connection provided many new problems that could be treated effectively using boundary integral equations, as the list of Frank's publications confirms with many papers in the annual QNDE reviews [8].

Frank retired from ISU in 2001. He and Mary Lou built a retirement home in northern Minnesota, dividing their time between there and Ames. He died in Ames on April 11, 2020.

2. The direct method

Let us return to Frank's thesis [R1] and his much-cited first paper [R2]. Both are concerned with two-dimensional plane-strain elastostatic boundary value problems, and both emphasise what would subsequently be termed the *direct method*, in which boundary integral equations are derived for unknown displacements u or tractions t (depending on the problem posed). These vector problems are treated in a manner

Fig. 3. Frank Rizzo as Chairman of the Department of Engineering Science and Mechanics, Iowa State University, 1988.

analogous to how Jaswon [3] had treated problems for Laplace's equation. This is distinct from so-called *indirect methods* in which the solution is written in terms of various layer potentials. As Jaswon noted, the basic direct methods can be used for mixed problems (where u is specified on part of the boundary and t is specified on the other part) and also for problems where the boundary consists of several disjoint pieces. (For more on Jaswon's contributions, see [6].) Frank also notes that, in principle, the basic approach can be used for three-dimensional problems.

In his thesis, Frank also describes classical indirect methods but "the clarity and physical appeal of always dealing with displacements and tractions directly is undeniable" [R1, p. 28]. He observes that the "presented methods involve the necessity of solving sets of boundary integral equations" and he describes a plausible numerical algorithm for solving such equations, paying attention to the presence of Cauchy principal-value integrals; however, no computations were performed. These had to wait until Frank had moved to the University of Washington.

The paper [R2] was submitted to the *Quarterly of Applied Mathematics* in November 1965, revised in August 1966, and published in early 1967. The initial submission was rejected "due to the absence of numerical results!" [2]. When Tom Cruse first met Frank at UW, "he was working with a graduate student [C.C. Chang] who could program in Fortran"...I had the distinct personal privilege to have been in the outer office when Frank came bounding in to declare 'It works!'" [2]. Frank has given more details [R43]:

> I regarded the computer at that time as a rather hostile beast, and Mr. Chang and I asked it to do only the most elementary and predictable things. Anything remotely sophisticated or unpredictable associated with the inevitable singularities we did analytically....[Then] on viewing our computational results, I believed much too strongly that our numerical success was a result of performing analytically as many 'element integrations' as we did. This belief, I think, slowed my subsequent confidence in and acceptance of the use of Gaussian quadrature with BEM. Such use is now, of course, the mainstay of all modern BEM procedures, but I was no help whatsoever in the initiation of this important development.

3. University of Kentucky

The first paper from the Rizzo–Shippy collaboration was published in 1968 [R4], the last in 1991 [R59]: "we both agree that we have accomplished more, and more enjoyably, working together, than ever would have been possible working separately" [R43]. In their early days, they "continued to use ad hoc computer codes (for two spatial dimensions and specific geometries), which were based on the assumption of

piecewise-constant boundary variables, analytical approximations, and exact methods for numerical solution" [7].

All that changed in 1975 with the papers by Lachat and Watson [4,5]. John Watson has given the background: "At the beginning of 1973 I started work on boundary elements....[My earlier] finite element programming had been most instructive in respect of shape functions, Gaussian quadrature and out-of-core simultaneous equation solution techniques. It seemed clear that the boundary elements should be isoparametric, with at least quadratic variation so that curved surfaces could be modelled accurately" [9]. Rizzo and Shippy "realized that these techniques represented an important advancement in boundary integral methods and decided to adopt them for our work. In all subsequent work we utilized isoparametric quadratic elements. (We were chagrined not to have been the first to adopt these techniques.)" [7].

Many applications followed: elasticity, thermoelasticity, axisymmetric structures, acoustics, elastic waves, and so on. See the list of publications in Section 7 for details.

Frank and Dave Shippy did not hesitate to draw upon the expertise of colleagues in the Department of Mechanical Engineering; Andy Seybert (acoustics), Bob Altenkirch (latterly president of the University of Alabama, Huntsville) and Dean Roger Eichhorn (latterly dean of engineering at the University of Houston), both in combustion. Frank also sought out Graeme Fairweather of the Department of Mathematics to provide advice on computational aspects of boundary integral equation methods. Fairweather's involvement led to his joint appointment in the Department of Engineering Mechanics in 1982. In 1986, working together, Frank and Graeme were instrumental in the establishment of the University of Kentucky's highly successful Center for Computational Science, funded by a National Science Foundation EPSCoR grant.

Also involved in the Rizzo–Shippy team were two outstanding PhD students, Benjamin Soenarko, who became a faculty member in the Department of Engineering Physics, ITB Bandung, Indonesia, and Mohsen Rezayat, who, since his graduation, has been Chief Solutions Architect at Siemens Digital Industries Software in Cincinnati, Ohio. Frank was held in high regard by graduate students in general, and these two in particular. According to Soenarko, "[Rizzo] was an excellent professor, smart, helpful and friendly", an opinion shared by many.

4. Iowa State University and hypersingular integral equations

Tom Rudolphi first became acquainted with Frank when he was a graduate student at UIUC around 1974/75. Frank gave a talk on boundary integral equations, and invited Tom to attend a short course at the University of Kentucky. Subsequently, he used Green's functions in his PhD work on fracture mechanics.

Some years later, Tom was on the faculty at Iowa State University (ISU) when, in 1985, the department chairmanship became open. He couldn't think of a better person to coax into that position than Frank. By that time, ISU had three other faculty members in the department who were using singular integral equations in their research in both fluid and solid mechanics (Les Schmerr, Marc Ingber and Ambar Mitra). Eventually, Frank accepted the position: in the summer of 1987 he and his family moved to Ames.

About one year before his move to ISU, Frank had met Paul Martin. They had a shared interest: crack problems. Problems involving cracks, screens, or other thin two-sided objects cannot be handled simply using standard direct boundary integral equations. The common feature of such problems is the occurrence of the normal derivative of a double layer potential or analogous vector quantities. Once these are interpreted, rigorously, as hypersingular integrals, hypersingular integral equations appear quite naturally. The associated analytical work, and the development of computational algorithms, gave Frank much satisfaction.

One of the application areas where hypersingular formulations found root was in wave scattering in acoustics and solid material elastodynamics. Wave scattering in solids was also of much interest in the ISU Center for Nondestructive Evaluation (CNDE) where boundary integral equations with strong singularities were used in scattering from volumetric inclusions (defects) in elastic solids, as well as from cracks and screens.

The ISU group (faculty, students, postdocs and visitors) made great progress in bringing hypersingular formulations into a practical methodology of computation. See the papers listed in Section 7 to get an idea of the range of applications as well as theoretical developments.

5. Minnesota

Frank retired from ISU in 2001. He and Mary Lou built a house in northern Minnesota on the shores of Lake Elbow near the town of Orr. Frank's love of Minnesota was ignited when he was 15 and an uncle took him on an organized fishing trip to Tuscarora Lodge in the northern part of the state near the Canadian border. This venture turned out to be a life changing experience for him. As soon as he returned home, his sole desire was to return to the Lodge immediately, which he did for that and the subsequent nine summers, until his marriage to Mary Lou. Initially, he was a handyman at the lodge but over the years he developed into an experienced fishing guide. In the late 1960s, Mary Lou and Frank purchased Bearskin Lodge on the shores of East Bearskin Lake, the nearest town being Grand Marais, 25 miles away. In 1973, they sold the Lodge, retaining two of the ten cabins. These cabins became the base of a successful seasonal canoe outfitting business which they ran with the help of their children until the mid 1980s. Frank and family spent his sabbatical year 1978–1979 from the University of Kentucky at Bearskin Lodge. What he hoped would be an uninterrupted year of research became more of an unending challenge to survive the Minnesota winter.

6. Concluding remarks

In 1993, Frank received the ASME Worcester Reed Warner Medal. This is awarded to an individual for seminal contribution to the permanent engineering literature.

In 2004, Frank was the first recipient of the Frank J. Rizzo Award. This award was established by the International Association for Boundary Element Methods (IABEM) in 2004 to honor outstanding individuals and their research in the field of BIE/BEM. The main criterion in selecting the winner is based on the seminal and life-long contributions of the individual in research and education, with long lasting impact on the field.

These are formal recognitions of Frank's influence on the computational mechanics community. Beyond that, he was a positive influence on everybody who met him: he was a singular man.

Declaration of Competing Interest

None.

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