# Time-Domain Scattering
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## CORRECTIONS AND ADDITIONS

November 10, 2023 `tdbook-add.tex`

## Corrections

p. 127. Half way down, change “compressional flow” to “compressible flow”.

p. 163. Missing ) in (10.11): should be $(\mathrm{S}(\partial u/\partial n))(P,t)$.

p. 176. Last line: “§5.14], For” should be “§5.14]. For”

p. 184. Two lines below (10.81): $h = \frac{1}{3}T$ should be $h = \frac{1}{3}T/N$. Error found by M. Ganesh.

## Opening quotation for Chapter 1:

*The velocities of pulses propagated in an elastic fluid, are in a ratio compounded of the subduplicate ratio of the elastic force directly, and the subduplicate ratio of the density inversely; supposing the elastic force of the fluid to be proportional to its condensation.*

Proposition XLVII in Book 2 of Newton’s *Principia*, as translated from the Latin by A. Motte in 1729. In more modern terms, speed of sound = $\sqrt{\text{bulk modulus/density}}$.

## 1.3.4. A simple initial-boundary value problem

[Bottom of p. 16]: “where (1.62) . . . can be found.” See also [13, Appendix] and [20, eqn (25)].

## 2.6. Moving sources

[Bottom of p. 57]: For much more on electromagnetic fields generated by moving sources, see [50, Chapter 23].

## 4.6. Further problems and boundary conditions

[p. 80, just before §4.6.1]: Yet another variant of (4.16) has been studied in [6],

$$\frac{\partial u}{\partial n}(P,t) + \int_0^t g(t-\tau)\,\frac{\partial u}{\partial \tau}(P,\tau)\,\mathrm{d}\tau = f(P,t), \quad P \in S, \quad t > 0,$$

where $g$ is given (it could be a generalised function) and the integral term is recognised as a Laplace convolution, (1.76). The special case with $g(t) = t^{-1/2}$ is [32, eqn (1.13)]. For electromagnetic analogues, see [33].

## 4.8. Existence and uniqueness

[p. 89, just before §4.8.1] . . . book by Sayas [M742]; see also [7].

### 7.2.1. Dirichlet boundary condition [Scattering by a sphere]

[p .114, end of 2nd paragraph]: For numerical computation of $\theta_n$ and $\beta_{n,m}$, see [18].

### 7.2.4. Literature

[p. 120, end of section, "For elastodynamic scattering... [M623, §III.C]."] Bahari *et al.* [3] consider scattering by a thick fluid-filled spherical shell, and give a good review of the relevant literature.

### 7.4.1. Dirichlet boundary condition [Scattering by a sphere]

[Bottom of p. 123, add]: For other integral equations with kernels involving $P_n$, see [M259, eqn (49)] and [17, eqn (5-2)].

### 8.2. Scattering frequencies

[p. 133, 3rd paragraph]: "Numerical methods ...": add [37]

### 8.3. Boundary integral equations

[p. 135, 2nd paragraph]: "Boundary integral equations ...": add [29]

### 8.6. Singularity expansion method (SEM)

[p. 137, above quotation]: "...collaborators...M70]": add [47]

[Top of p. 138]: "Later papers include ...M73]": add [48].

[Bottom of p. 138]: Conventional SEM has been used for hydrodynamic problems, with frequency-domain software employed to compute natural frequencies [42, eqn (28)].

### 9.1.1. Kirchhoff's formula for bounded domains

[p. 146, just before §9.1.2, after $\delta/\delta n$.] For an early discussion, see [16, §III.6], especially [16, p. 179, eqn (35)].

### 9.1.7. Literature [on Kirchhoff's formula]

[Top of p. 150, with insertions:] ...has been translated [M485], [26] and analysed [M138], [24].

### 9.2. Kirchhoff's formula: a space-time derivation

Last line, p. 151, "...IBVPs [M693, M694]": add [34].

### 9.3.2. Moving surface $S(t)$

[p. 154, 2nd paragraph, about Ffowcs Williams–Hawkings, after [M345, Chapter 5] ]: For associated inverse problems, see [8].

[p. 156, end of §9.3.2]: Additional references: [23]

## 10.2. Integral equations: direct method

[p. 163, below (10.13)]:

Transmission problems (Section 4.6.2) can be reduced to coupled pairs of time-domain boundary integral equations (TDBIEs) [M549, eqn (2)], [10, eqns (5) and (6)], much as they are in the frequency domain [M599, §6.2]. See also [21, §8.2]. It turns out to be advantageous to use a time derivative of standard TDBIEs, just as (10.13) is related to (10.12):

> Ideally, one could remove the time derivative and still solve the resulting equation. However, as explained in [M377, M378, M721, M722], for the solution of TDBIEs to be stable, basis and testing functions should be selected from appropriate Sobolev spaces. ... Following the analysis in [M721, M722], the testing function for TDBIEs without the time derivative should be selected as the time-derivative of the Dirac delta function. This means that a marching method which relies on point-testing in time (i.e., testing with Dirac deltas) does not yield a stable solution when it is used to solve TDBIEs without the time derivative. [10, p. 1066]

For acoustic scattering by inhomogeneous obstacles, see [21, §8.1].

## 10.3.1. Basic time-stepping method

[p. 166, just before §10.3.2, add:] For the damped wave equation (1.69), see [44].

## 10.3.2. Instabilities and remedies

[p. 166, 2nd paragraph]: "Frequency-domain integral equations ... M318]": add [10, 46, 45].

[p. 167, line 1]: "B-splines [M717, M223]": add [46, 45].

[p. 167, 3rd paragraph]: Pölz & Schanz [M702]... variable; see also [36].

## 10.4.2. Application [of CQM] to time-domain BIEs

[p. 170, 4 lines below (10.39)]: "For overviews ... M390]": add [27]. Then, after "For some criticism, see [M19, §2.2.1]": For many details and applications, see the book by Banjai & Sayas [7].

[Top of p. 171, after "CQMs with BDF 2 ... [M706]."] For mixed boundary conditions, see [40]. For the damped wave equation (1.69), see [5].

[Next paragraph]: "For some applications of Runge–Kutta ... M391]", [7, Chapter 5] and [22].

## 10.5. Electromagnetics

[p. 176, above the quotation]: after "... scattering [M90, M399]." Add "See also the 1987 review by Miller [31]." Then "Subsequent work ... M219]", add [30, 38, 39].

[p. 176, below the quotation]: Nevertheless, explicit marching-on-in-time methods continue to be developed [M845, M171], [9].

"Time-domain versions ... M169]." Add [49]. [Further down]: "For fast methods ... " Add [43].

[Last paragraph]: After "M158]", add [14]. Then, change "For a review, see [M550]." to "For reviews, see [M550], [15] and [7, Chapter 6]."

## 10.6. Elastodynamics

[p. 177, line 3]: "For three-dimensional ... M679]": add [4, 12, 25, 41, 1].

[p. 178, penultimate paragraph]: "Another option ... [M281]." Add [51]

[p. 178, end of last paragraph]: See also [51]. The space-time energetic Galerkin method of Aimi et al. [M9, M10] has been extended to elastodynamics [2].

### 10.7.1. Clément's equation

For more on Clément's approach, see [11].

## 10.9. Use of Fourier transforms

[Bottom of p.183]: "can be effective [M487]": add [28].

### 10.10.3. Comments and literature [on cracks and screens]

[p. 192, end of 2nd paragraph]: For more on scattering by thin flat screens, see [21, §9.2].

[Bottom of p. 192]: Plane-strain problems have also been solved using boundary integral equations in the Laplace-transform domain [19, 35].

## References

p. 199. Reference [M138] is reprinted in [24], pp. 63–123.

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