Internal Erosion of Embankments: A Review and Appraisal

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Internal Erosion of Embankments: A Review and Appraisal

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ABSTRACT

Internal erosion (IE) refers to any process by which soil particles are eroded from within or beneath a water retaining structure. As half of embankment dam failures are caused by IE, it is an issue of major concern. Approaches for designing against IE have been well established for over 50 years; however, a large fraction of existing embankments are still vulnerable to erosion. This paper presents a review of the four distinct types of IE, the general process of IE failures from initiation to breach, methods for designing against IE, and the current state of the art with regard to evaluating IE potential. For each IE mechanism, deficiencies in the current state of the art are identified, and new research directions are proposed for advancing engineering practice.

INTRODUCTION

Internal erosion (IE) refers to any process by which soil particles are eroded from within or beneath a water retaining structure. IE is a particularly dangerous process as it gradually degrades the integrity of a structure in a manner that is often completely undetectable. Further, 46 percent of all historical embankment dam failures have been attributed to IE, making it one of the greatest risks associated with embankment dams, second only to overtopping related failures (Foster et. al 2000).

IE can be subdivided into four distinct erosion mechanisms: concentrated leak erosion, backward erosion piping, internal instability, and contact erosion (Bonelli 2013; ICOLD 2015). In general, the methods of designing embankments against IE are the same for all four types of IE. However, in the case of existing embankments thought to be vulnerable to erosion, evaluating the potential of IE leading to failure is entirely mechanism specific. For this reason, it is crucial that engineers be familiar with the four IE mechanisms and the corresponding methods of analysis. The following sections provide an overview of IE mechanisms, the state of the practice for designing against IE, and the methods available for analysis of IE potential with emphasis placed on deficiencies in the state of the art.

INTERNAL EROSION MECHANISMS

Early efforts at assessing IE lumped all erosion-related failures into a single category (Bligh 1910; Lane 1935). It was not until Terzaghi and Peck (1948) separated IE failures into two categories that the different types of erosion began to be distinguished. More recently, IE has been subdivided into four broad categories based upon the mechanics of the erosion processes (Fell and Fry 2007). The four IE mechanisms are: concentrated leak erosion, backward erosion piping, internal instability, and contact erosion (Bonelli 2013; ICOLD 2015). Figure 1 illustrates all four types of erosion as they might occur on an embankment dam. The four mechanisms are defined as follows.

1. Concentrated leak erosion: Concentrated leak erosion refers to any process in which water flows freely through an open space, eroding the soil along the boundary of the opening. Examples of concentrated leaks include flow through desiccation cracks, flow through gaps along penetrations through embankments (e.g., conduits, walls, utilities), and flow through animal burrows. Concentrated leak erosion is by far the most dangerous IE mechanism, accounting for the majority of IE incidence and failures (Fry 2016, Foster et. al 2000, Richards and Reddy 2007).
2. Backward erosion piping (BEP): BEP is a process by which open erosion channels (often called "pipes") progress "backwards" through an embankment foundation as illustrated in Figure 1. For BEP to progress completely beneath an embankment, seepage forces must be high enough to initiate erosion and propagate the pipe, the flow in the pipe channel must be sufficient to transport the particles to the downstream "sand boil", and an overlying material must provide "roof support" for the erosion channel to remain open. BEP typically occurs in foundation sands where the conditions for BEP development are easily met. However, it can also occur in other locations, e.g., in loose fill placed along conduits. While Richards and Reddy (2007) estimate BEP accounts for only one-third of IE-related dam failures, it is a prevalent IE mechanism along levee systems as evidenced by the more than 1,000 sand boils discovered along the Mississippi River in 2011 alone (USACE 2012).
3. Internal instability: Internal instability refers to any process by which seepage is able to selectively erode the finer grains of soil from a matrix of coarser grains as illustrated in Figure 1. Internal instability can be further subdivided into suffusion (erosion without volume change) and suffosion (erosion with volume change) (Fannin and Slangen 2014). Despite being one of the least dangerous IE mechanisms (Fry 2016), extensive research has been conducted on internal instability, largely due to the visible movement of soil that it has caused on many embankment dams (Ronnqvist and Viklander 2016).
4. Contact erosion: Contact erosion occurs when flow through a coarse-grained material erodes an adjacent material of smaller grain size (Figure 1). Contact erosion is a significant concern in France where silt levees have been placed upon gravel foundations along the rivers (Bonelli 2013; Fry 2016). The process appears to progress slowly and requires relatively high velocities for substantial erosion to occur.

As readily observed from the four mechanism descriptions, the physics involved in each erosion process is quite different. Internal instability and contact erosion depend largely on flow through porous media. Concentrated leak erosion depends almost entirely on the hydraulics in channels and pipes, as well as the surficial erodibility of the soil. Backward erosion piping depends equally on flow through porous media, flow through open channels, and concepts from sediment transport. Despite these significant differences, a single approach can be used to design against all internal erosion mechanisms.

DESIGNING EMBANKMENTS TO PREVENT INTERNAL EROSION

For any IE mechanism to develop towards failure, two conditions must be met: (1) eroded soil particles must be able to move downstream and (2) sufficient flow must exist to cause erosion. The strategy for designing embankments to prevent IE is to eliminate both conditions through the use of modern filters and drains. Historically, however, the concept of creep ratios has been promoted for design against IE. In the following sections, the shortcomings of creep ratios are mentioned, followed by the design philosophy for filters and drains.

Creep Ratios – A Historical Approach

The concept of creep ratios must be discussed as it was suggested by classic soil mechanics books for design against IE (often called "piping" in this context) (Terzaghi and Peck 1948; Taylor 1948; Harr 1962). More recent textbooks have recognized the flaws inherent in the concept of creep ratios and have excluded the topic altogether, e.g. Briaud 2013.

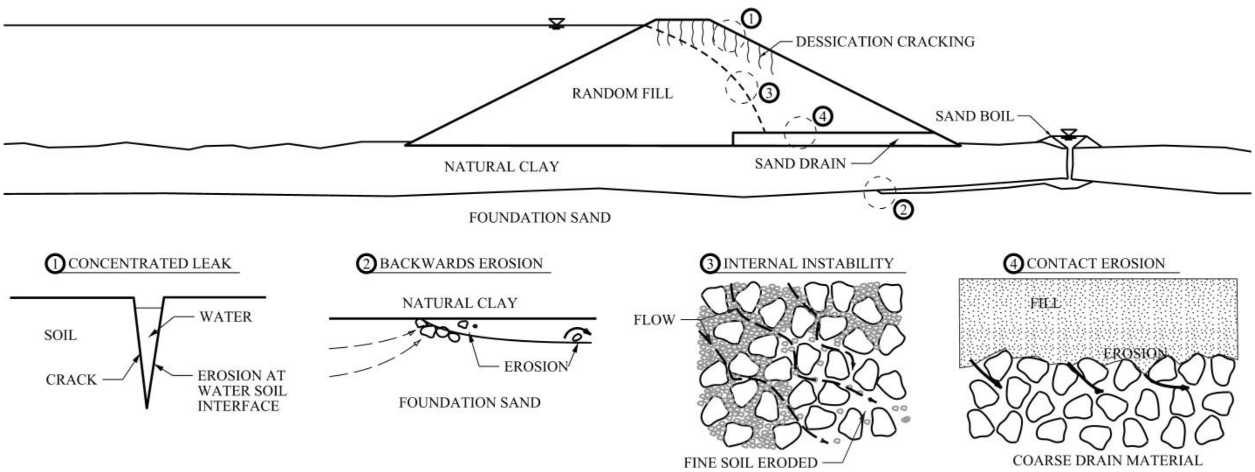


FIGURE 1. Illustration of the Four Types of Internal Erosion.

The creep ratio for a particular dam, C_r, was originally defined as

$$C_r = (L_h + L_v) / H \quad \text{(Eq. 1)}$$

where L_h and L_v designate the horizontal and vertical seepage lengths along the contact between a dam and its foundation, respectively, and H designates the differential head that can be safely retained by the structure (Bligh 1910). Eventually, the effectiveness of vertical cutoffs was recognized, and the weighted creep ratio, $C_w = (L_h / 3 + L_v) / H$, was developed (Lane 1935). Design values for weighted creep ratios based upon soil type were developed empirically by evaluating 251 concrete and masonry dams, of which only 21 had failed (Lane 1935). As the recommended creep ratios were based on concrete and masonry dams with highly variable foundation configurations, the differences between embankment foundations and the dam foundations Lane examined should be carefully considered before applying in practice.

Even at the time of publication, Casagrande (1935) recognized that this empirical approach was substantially in error, resulting in both over- and under-conservative designs due to the variability in foundation conditions between projects. From a modern perspective, the creep ratios are of limited value due to: (1) the combination of all IE failures into a single category, (2) temporal non-stationarity of dam failure statistics because of improvements in design and construction practice since 1935 (ICOLD 1995), and (3) the ability to entirely prevent IE through properly designed filters and drains. Designers should not use creep ratios to design future embankment dams against IE, and instead should rely on filters and drains to prevent internal erosion.

Filters and Drains

There have not been any documented IE failures of embankment dams containing adequate filters and drains (Fry 2016). To be considered adequate, a filter and drainage system must protect the entire embankment with filters and drains. In cases with positive cutoffs into relatively impervious foundations, this is accomplished by ensuring the entire dam core is protected on the downstream side with filters and drains. For cases in which a positive cutoff is not economically possible, adequate drainage of the foundation through application of blanket and toe drains can be used to control seepage (Figure 2). A filter must also meet particle retention and permeability requirements as documented by FEMA (2011). If a filter and drain system meets these requirements, particle movement through the filter will not be possible, and the hydraulic conditions for IE to occur will not develop.

To understand the impacts that filters and drains have on the four IE mechanisms, consider the modified embankment dam section illustrated in Figure 2. If a concentrated leak developed in the top of the dam as shown in Figure 1, the chimney filter and drainage system shown in Figure 2 would capture the leakage through the dam, preventing the leak from exiting the crack. Further, any particles eroded upstream of the filter would be stopped by the filter. This would ultimately lead to the crack in the embankment being sealed due to material accumulating on the upstream filter face. With regard to BEP, the downstream toe drain will reduce the seepage forces at the downstream embankment toe. In the event the toe drain is inadequate and BEP initiates, the hydraulic gradient downstream of the toe drain would be sufficiently low to prevent progression of a pipe upstream. Considering the case of internal instability, it is immediately evident that any mobile soil particles upstream of the filter would be stopped at the filter. Mobile particles downstream of the chimney filter would be unlikely to move substantially as the chimney drain should prevent seepage from passing through the downstream portion of the embankment. Lastly, with regard to contact erosion, if the materials all meet modern filter criteria for particle retention, it is not possible for contact erosion to occur.

From the example presented, it is demonstrated that filters and drains can be used to prevent all four internal erosion mechanisms. For further details on the development of standards pertaining to filters and drains, the interested reader is referred to Redlinger et al. (2016). Detailed guidance on filter design is provided in FEMA (2011).

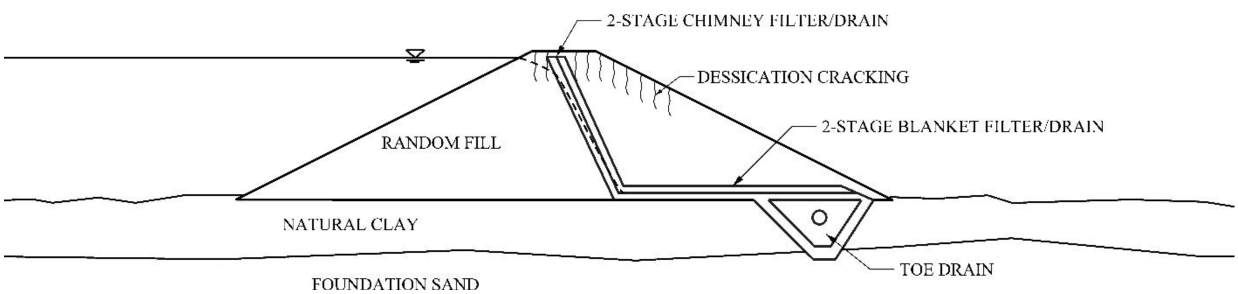


FIGURE 2. Schematic of a Modern Filter and Drain System to Prevent IE.

RISK ANALYSIS FOR INTERNAL EROSION

While filters and drains provide a means to design new embankments for IE, it is estimated that more than half of existing large embankment dams in the U.S. do not meet modern standards (Redlinger et. al 2016). Further, for many small embankments such as levees, it is often not economically feasible to include designed filters and drains. All of these embankments remain vulnerable to IE and pose risks to society. This section discusses the state of the art for assessing the IE risks associated with each IE mechanism. An appraisal of current gaps in the assessment process was made, and recommendations for future research are provided.

Generic IE Event Tree

For all IE mechanisms, an event tree approach is used to develop risk estimates (USBR and USACE 2015; Paté-Cornell 1984; Whitman 1984). A generic IE event tree is illustrated in Figure 3. In practice, each of the four events shown may be subdivided into numerous events leading to event trees with dozens of nodes. However, for the sake of simplicity in this discussion, the event tree has been limited to the major phases of IE development. Initiation of IE refers to the initial movement of soil particles. Once erosion initiates, it must progress through the continual removal and transport of additional soil particles. If erosion is able to progress, the developing void must be able to remain open as it continues to enlarge (continuation). If the soil cannot support a large void, it may collapse and limit the potential for failure. Lastly, if erosion continues to progress and the reservoir is breached, the eroded void must enlarge sufficiently to allow uncontrolled breach or release of the reservoir.

Initiation ➡ Progression ➡ Continuation ➡ Breach

FIGURE 3. Basic Nodes of an IE Event Tree.

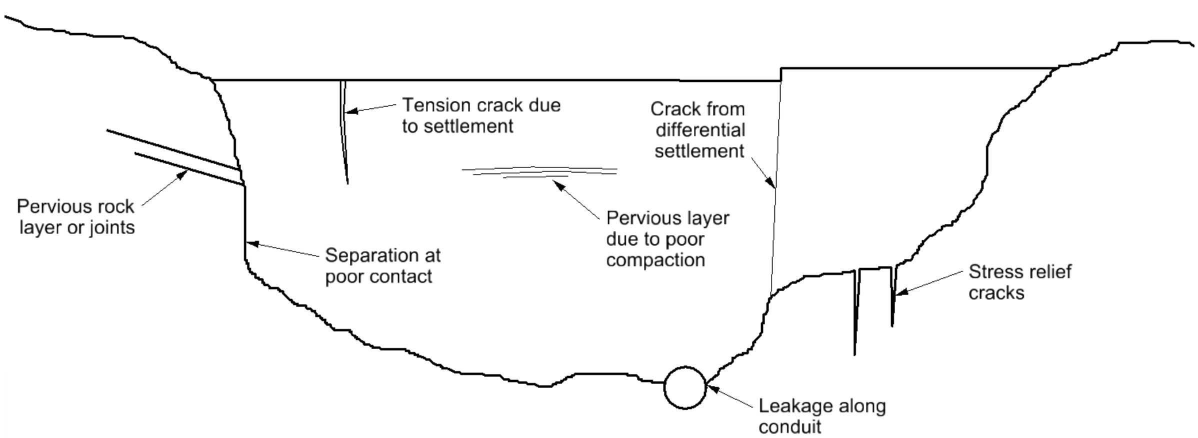
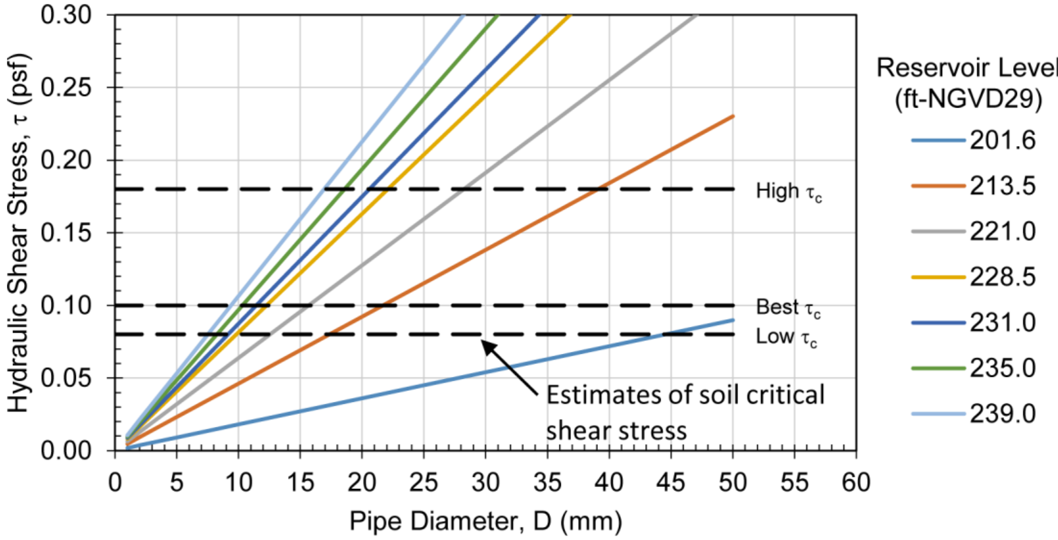


FIGURE 4. Locations of Potential Leaks in Embankment Dams (after USBR and USACE 2015).

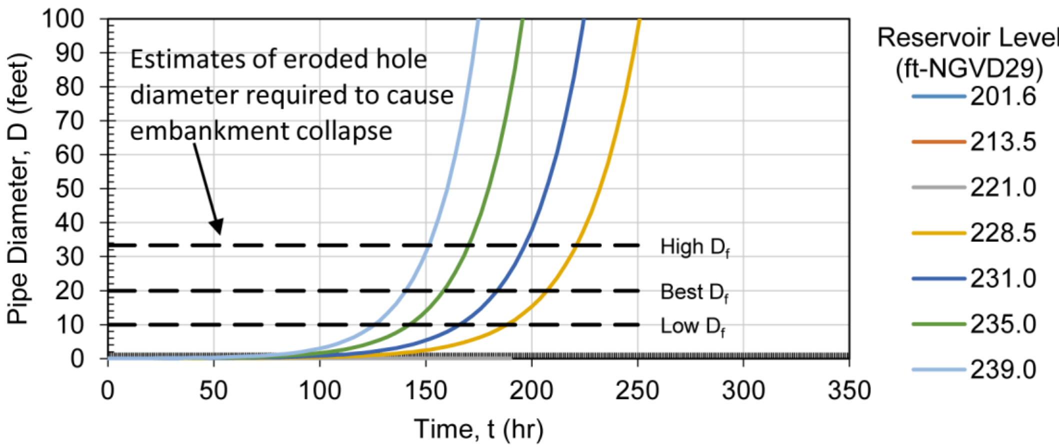
For quantitative probability estimates, the values assigned to each event tree node are subjectively estimated through a combination of judgment and analysis. Discussion of the role of engineering judgment in risk analysis is far outside the scope of this paper; for further details, the reader is referred to Vick (2002). In the following sections, the state of the art with regard to *quantitative analyses* solely conducted for evaluation of each IE mechanism is discussed.

Concentrated Leak Erosion

Initiation of concentrated leak erosion requires that a crack or defect exist such that concentrated seepage flow can pass through the embankment. Estimates of crack/defect size, shape, and location are made based on past experience and engineering judgment. From case histories, the locations and conditions likely to cause a soil to crack or separate from structural features are qualitatively understood. Figure 4 illustrates some common locations where cracks/defects are often observed. Soil properties and field observations are then evaluated by engineers, and a range of crack locations, shapes, and sizes are proposed for evaluation of concentrated leak erosion. As estimation of the crack characteristics is judgment based, it will not be discussed any further. Development of models to aide in crack prediction is currently an active area of research (Fell 2016).



a. Estimating Shear Stress on Defect Walls



b. Evaluation of Defect Enlargement for Constant Reservoir Levels

FIGURE 5. Sample Computations for Concentrated Leak Erosion (USBR & USACE 2015).

To evaluate the probability of concentrated leak erosion developing to failure, sensitivity analyses are typically conducted to determine: (1) how large a defect would need to be for erosion to initiate and (2) how fast would a defect enlarge if it indeed existed. For each crack/defect location considered probable, the shear stress on the defect wall (τ) is estimated as:

$$\tau = \gamma_w (\Delta H / L) A / P_w \qquad \text{(Eq. 2)}$$

where γ_w is the unit weight of water, ΔH is the hydraulic head difference across the defect, L is the length of the defect, A is the cross-sectional area of the defect, and P_w is the wetted perimeter of the defect (Wan and Fell 2004). To develop probabilities of erosion initiating in the defect, an estimate of the distribution of critical shear stress, τ_c, at which erosion initiates is required. Probability distributions can be assigned to all variables in Eq. 2, and the probability of erosion initiating is simply given by $P(\tau > \tau_c)$.

If it appears likely that erosion will initiate, it is then necessary to consider how fast the defect may enlarge. The rate at which erosion occurs along the walls of the crack/defect ($\dot{\varepsilon}$, ft/s) is estimated by:

$$\dot{\varepsilon} = k_d(\tau - \tau_c) \qquad \text{(Eq. 3)}$$

where k_d ($ft^3lb^{-1}s^{-1}$) is the soil erodibility. Simple numerical methods, e.g., Runge-Kutta time integration, can then be used to estimate the evolution of the defect for anticipated hydrologic loadings. Figure 5 illustrates sample computations demonstrating the influence of reservoir level and initial defect size on the shear stress estimates (5a), and the influence of reservoir stage on the rate of pipe enlargement (5b) for an assumed defect shape.

The above computations are focused solely on the initiation and continuation of erosion, assuming a defect exists. As shown, these analyses are quite sensitive to the defect dimensions and location. Further, the soil properties k_d and τ_c have been shown to vary 4 to 5 orders of magnitude for typical embankment materials (Wahl et. al 2009). Despite the significant uncertainty surrounding concentrated leak erosion computations, a few key observations can be made. First, the enlargement of defects is a highly non-linear process, increasing in rate of development with increasing defect size (Figure 5b). This observation correlates well with observations from actual failure case histories, e.g., Teton Dam. Second, if a defect size is reasonably known, assessing the initiation of erosion is the most reliable computation in the entire analysis as it contains the fewest number of assumptions (relying on only τ_c). Perhaps because of these observations, Fell (2016) suggested the following research needs regarding concentrated leak erosion.

- Research into predicting crack/flaw location and dimensions from numerical modeling of embankment deformations is needed.
- Research into estimating τ_c of embankment materials is needed. Specifically, understanding the influence of soil classification, degree of saturation at compaction, and changes over time are needed. Further, an understanding of the variation in τ_c measurements from various erosion tests (jet erosion test, hole erosion test, erosion function apparatus test) is needed.

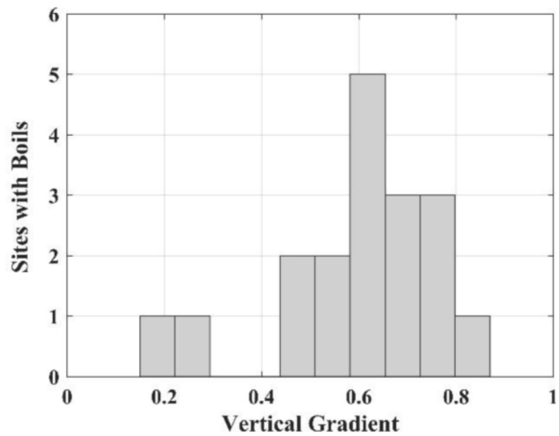
Improved estimates of defect characteristics and τ_c will increase the reliability of concentrated leak erosion probabilities greatly, allowing failure modes for many sites to be ruled out altogether. Future research for concentrated leak erosion should focus on these aspects of the erosion process.

Backward Erosion Piping (BEP)

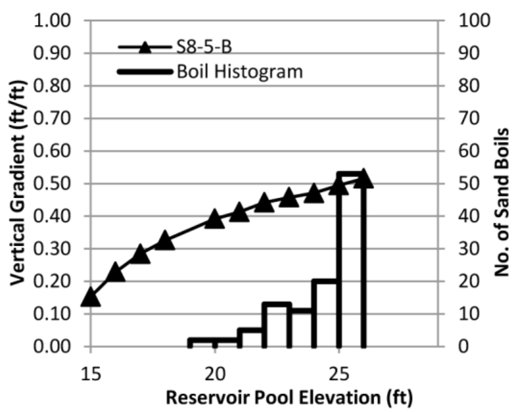
Analyses of BEP in the U.S. has historically focused only on the initiation node in the event tree. Initiation has been assessed by considering the vertical critical gradient, i_c, given by:

$$i_c = (1-n)(G_s - 1) \qquad \text{(Eq. 4)}$$

(Terzaghi and Peck 1948), where n designates the soil porosity and G_s designates its specific gravity. Values of i_c range from 0.8 to 1.1. It has been demonstrated that i_c does correspond to particle movement in cases with uniform seepage through homogenous soil (Terzaghi 1929). However, measurements of vertical gradients in the vicinity of sand boils indicate that particle movement typically initiates at gradients far below i_c (Figure 6).



a. Along Mississippi River Levees (Turnbull and Mansur, 1961)



b. Along a Ring Dike in Florida (Huzjak et al. 2016)
FIGURE 6. Field Measurements of Vertical Gradients near Sand Boils.

An explanation for the apparent discrepancy between observation and theory is offered by the fact that sand boils typically occur in areas with thin confining layers where defects, e.g., root holes, fence post holes, animal burrows, etc., are likely to exist. With seepage flow concentrated at these defects, soil particles can readily be moved at ambient gradients far below i_c. Unfortunately, if initiation of BEP is truly controlled by local variations and defects in the soil, reliable prediction of BEP initiation from mechanistic approaches is no longer possible, and empirical methods must be relied upon. Further research is needed to develop standard methods for converting observations of sand boils in the field to estimates of probability of BEP initiation.

Past experience has demonstrated that initiation of BEP is extremely likely to occur along rivers during flood events (Glynn and Kuszmaul 2010; USACE 2012, van Beek et al. 2013). However, failure of levees due to BEP is relatively infrequent , e.g., no failures occurred due to BEP at levees along the Mississippi River in 2011 despite the occurrence of 1,000 plus sand boils. This indicates that BEP is often controlled by the progression of erosion. Two approaches exist for evaluating the progression of BEP, i.e., the method proposed by Sellmeijer (1988, 2006) and the method proposed by Schmertmann (2000).

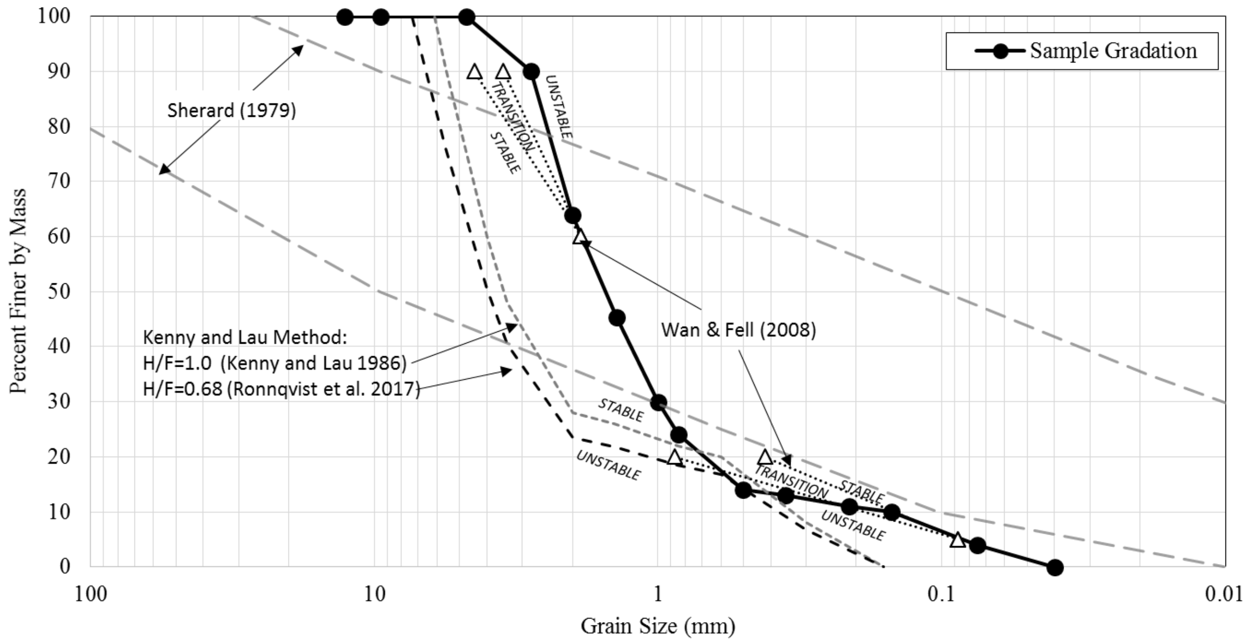
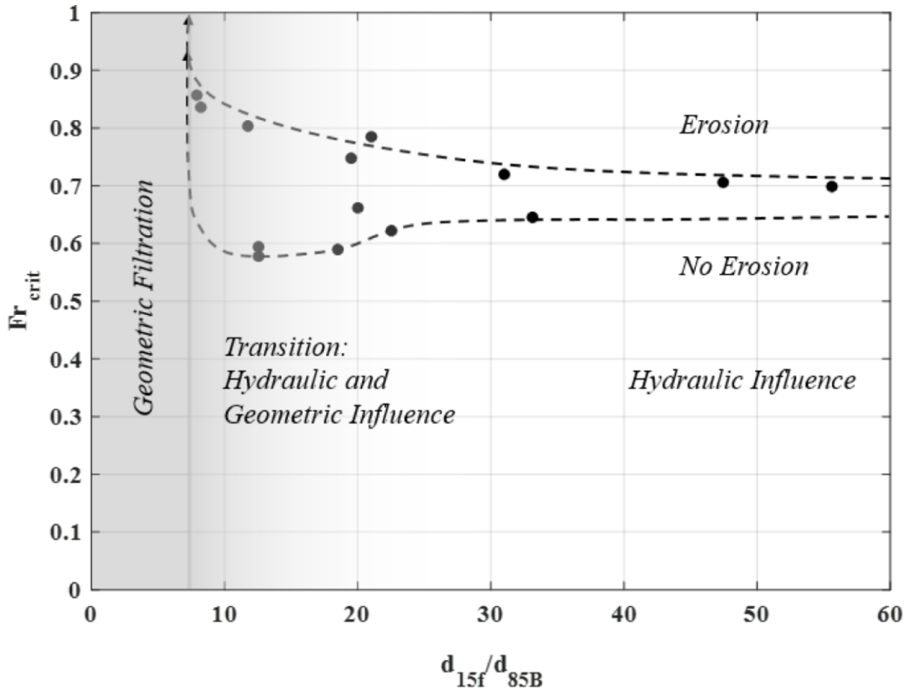


FIGURE 7. Comparison of Gradation Shape Analysis Techniques for Assessing Internal Stability of Soils.

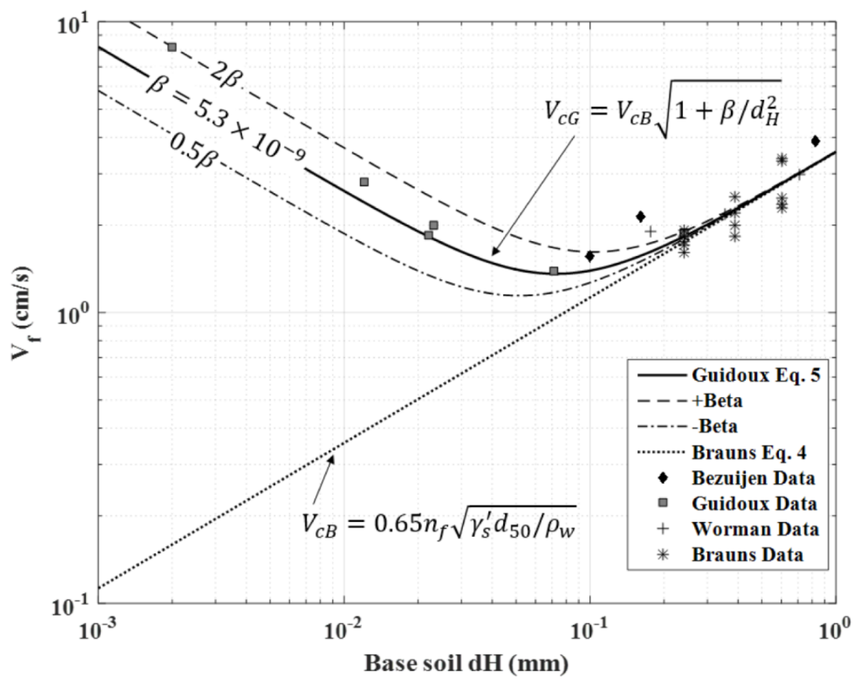
Sellmeijer (1988, 2006) developed a 2D BEP model that couples groundwater flow solutions with laminar pipe flow equations to model the overall hydraulics of BEP. The model assumes that BEP progression is controlled by the transport of soil grains in the erosion pipes, i.e., the hydraulic conditions in the pipe are unique during erosion. As demonstrated by van Beek (2015), BEP progression is actually controlled by the conditions at the upstream tip of the erosion pipe. Because of this assumption and the 2D limitations, the model is only applicable to a select set of problems.

Schmertmann (2000) developed a method for assessing BEP progression based on the ambient hydraulic gradient along the erosion path, i_p. The method is based upon estimates of i_p obtained from laboratory piping tests, from which it was found that the critical value of i_p for erosion progression is given by $i_{pc} = 0.0 + 0.183(C_u - 1)$ where C_u designates the coefficient of uniformity of the soil. Application of this method requires that i_{pc} for the laboratory conditions be adjusted for influential factors, e.g., problem geometry and soil properties, to match field conditions. In practice, the ability to reliably make these adjustments has proven very difficult.

Neither of the methods presented above has been shown to suitably predict BEP progression in a general sense. Experimental research must be conducted to determine precisely what local hydraulic conditions near the upstream end of erosion pipes lead to further erosion. Numerical models must then be developed that capture the entire BEP process in sufficient detail, i.e., groundwater flow, pipe flow, sediment transport, and three-dimensional nature, to discern the differences between safe and unsafe conditions.



a. Influence of Geometry and Hydraulic Conditions on the Critical Froude Number for Erosion (Fr) (Brauns 1985).



b. Critical Velocity for Contact Erosion in Region of Hydraulic Influence (adapted from Guidoux et al. 2010).

Figure 8. Geometric and Hydraulic Considerations for Evaluating Contact Erosion.

Internal Instability

Analysis methods developed for assessment of internal instability have focused entirely on the conditions required for initiation of particle movement. All analysis techniques developed can be summarized as “gradation-shape analysis” techniques that relate quantitative characteristics of a soil gradation curve empirically to either field performance of embankment dams or laboratory tests designed to assess internal instability (Li and Fannin 2008; Ronnqvist and Viklander 2016; Wan and Fell 2008). Sherard (1979) recognized that broadly graded soils tended to be internally instable and proposed, based on performance of embankment dams, that soils with gradations in a certain range (Figure 7) are likely to be unstable. Kenny and Lau (1986) more closely examined the shape of the gradation curve through a recursive application of filter criteria in which a fine, mobile fraction (F) is compared to the mass fraction capable of filtering the fine fraction (H). This results in a continuous boundary based on the gradation curve that designates internally unstable soils if crossed. Ronnqvist et al. (2017) found the Kenney and Lau boundary to be overly conservative and suggested a modification resulting in a slight change (Figure 7). Wan and Fell (2008) noticed that internally unstable soils tend to have gradation curves with a steep coarse fraction and a flat fine fraction. As such, they proposed a criterion that examines the slope of the gradation curve at two points as illustrated in Figure 7. While different, all of these approaches are simply assessing the shape of the gradation curve. For the example gradation curve shown in Figure 7, it is easily seen that all of the techniques would indicate the example gradation is potentially unstable.

While the soil gradation shape analysis techniques can be used to determine if an embankment is likely to contain internally unstable materials, they do not allow the probability of failure to be determined as only the initiation node of the event tree has been informed by the analysis. Recent research has begun to evaluate the temporal evolution of IE due to internal instability (Nguyen et. al 2012) as well as the influence of IE on the mechanical properties of soils (Ke and Takahashi 2012). Further, numerous approaches for simulating the consequences of erosion in embankment dams have been proposed, e.g., Liang et al. 2017. While this recent research is beginning to look into progression of erosion, no predictive models exist to date that have been demonstrated to reliably predict the evolution of erosion over time.

Perhaps one reason for the lack of such predictive models is the variability associated with erosion experiments investigating internal instability. Most investigations into IE progression have focused on studying the temporal evolution of erosion. Erosion is a highly-stochastic process dependent on the “weakest” link in a soil sample. Because of this, it is extremely difficult to obtain repeatable test results in the sense of the precise temporal behavior of erosion. In the authors’ opinion, future research can overcome these obstacles by focusing on limiting behavior rather than the temporal process of erosion. That is, research must be conducted to determine if there is a unique limit to the amount of erosion that may be expected for a particular soil under a given hydromechanical load. By focusing on the limiting behavior rather than the temporal behavior, much of the uncertainty with soil testing may be eliminated, allowing for a more reliable, repeatable quantification of the erosion process. If a unique limit to mass erosion exists for particular hydromechanical states, erosion limit state functions can be defined experimentally for all pertinent soil properties, e.g., volume, density, permeability, stiffness, and strength that describe the impacts of IE on embankment dams under the worst of conditions. This information can then be used to assess the extent of erosion progression as well as the structures mechanical response to the anticipated erosion. Research focused on obtaining this type of information will potentially allow geotechnical engineers to determine if embankment dams containing unstable materials are safe for continued use.

Contact Erosion

The analysis of contact erosion has, similar to internal instability, focused solely on whether or not erosion will initiate. Two conditions are needed for contact erosion to initiate (Bonelli 2013).

1) Geometrical condition: Pores of the coarse soil layer have to be sufficiently large to allow particles to pass through them.
2) Hydraulic condition: Seepage flow velocity has to be sufficient to detach the soil particles and transport them.

The geometric condition is assessed by comparing the particle size of the coarser soil for which 15 percent is finer (d_{15f}) to the particle size of the finer soil for which 85 percent is finer (d_{85B}). In general, situations for which the ratio d_{15f} / d_{85B} is less than 8 do not meet the geometric condition and are unlikely to experience contact erosion (Bertram 1940, Sherard et al. 1984). If the geometric condition is met for contact erosion to occur, the critical Darcy velocity in the coarse material, V_c, at which erosion of the finer soil is expected, must be estimated. As shown in Figure 8a, a transition zone exists as d_{15f} / d_{85B} approaches the geometric condition in which V_c is dependent on both geometric and hydraulic conditions. Once d_{15f} / d_{85B} becomes greater than 30, purely hydraulic conditions control the erosion, and V_c can be estimated from Figure 8b. For erosion of sand in contact with gravel, Brauns (1985) found that V_c is given by

$$V_c = 0.65 n_f \sqrt{\gamma'_p d_{50} / \rho_w} \qquad \text{(Eq. 5)}$$

where n_f, γ'_p, d_{50}, and ρ_w represent the gravel porosity, specific weight of sand particles, median diameter of the sand, and density of water, respectively. Guidoux et al. (2010) found that V_c increases for fine-grained soils due to particle cohesion, and proposed modifying Eq. 4 to:

$$V_c = 0.65 n_f \sqrt{\gamma'_p d_H / \rho_w (1 + \beta / d_H^2)} \qquad \text{(Eq. 6)}$$

where β is simply a constant representing particle cohesion and d_H is the effective grain diameter. A comparison of Eq. 5 and Eq. 6 is given in Figure 8b. Similar to internal instability, the current state of the art for assessing contact erosion does not allow for the probability of failure to be determined from analysis. The above computations inform the initiation node on the event tree, but the rest of the event tree must be evaluated through judgment. Further research is needed to quantify the rate of contact erosion. The rate of soil erosion is critical information for assessing the probability of failure.

CONCLUSIONS

The four types of internal erosion (IE) were reviewed along with the design strategies to prevent IE through the use of filters and drains. Unfortunately, many embankments are not designed to modern standards, and risk analyses must be used to assess the safety of those existing dams with regard to IE. Significant research is needed to develop analysis methods for assessing all nodes in the IE event tree. Research regarding erosion progression is needed for the mechanisms of BEP, internal instability, and contact erosion; research regarding defect and material characteristics are needed for concentrated leak erosion. Until the state of the art is advanced in these areas, risk analysis for IE will continue to depend almost exclusively on engineering judgment.

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